HOME ASSIGNMENT

B.Sc. 2nd Semester (CBCS)
Sub: Mathematics (Honours)
Paper: MAT-HC-2016 (Real Analysis)
Total Marks: 50

Date of Submission: On or before 8th August 2020

1. Answer the following questions: $1 \times 7 = 7$

(a) If β a limit point of a sequence (S_n) then there exists a subsequence (S_{n_k}) of (S_n) which converges to β. (write true or false)

(b) Define a Cauchy sequence.

(c) Write a subsequence of natural numbers.

(d) What is the range of the sequence if $f: \mathbb{N} \to \mathbb{R}$ defined by $f(n) = x_n = k$, where k is an arbitrary constant.

(e) Any finite set is bounded. (Write true or false)

(f) If $b \neq 0$, $d \neq 0$ then $(ab^{-1})(cd^{-1}) = (ac)(bd)^{-1}$ $\forall\, a, b, c, d \in F$ (Write true or false)

(g) Is the sequence $(1, -1, 1, \cdots\cdots (-1)^{n+1}, \cdots\cdots)$ monotonic?

2. Answer the following questions: $2 \times 5 = 10$

(a) Find the values of x and y, if $S = \{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\}$, $\inf S = x$ and $\sup S = y$.

(b) Prove that the sequence $(\frac{1}{n})$ is a Cauchy.

(c) Show that $\lim \left(\frac{3n+2}{n+1}\right) = 3$.

(d) Solve $(2x+1)(3x-5) > 0$.

(e) If $c \geq 0$ then show that $|a| \leq c$ if and only if $-c \leq a \leq c$.

3. Answer the following questions: (any two) $4\times2=8$

(a) If a and b are any two real numbers, then prove that $|a-b| \geq ||a|-|b||$

(b) If $x>0$ and $a<b$ then prove that $a<\frac{a+xb}{1+x}<b$

(c) Prove that a necessary condition for convergence of an infinite series Σu_n is $\lim_{n\to\infty} u_n = 0$

4. Answer the following questions (any five) $5\times5=25$

(a) If (a_n) and (b_n) be two sequences such that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then prove that $\lim_{n\to\infty}(a_n b_n) = ab$.

(b) State and prove Bolzano-Weierstrass theorem for sequence.

(c) Prove that every Cauchy sequence is bounded. Is the converse true? Give reason.

(d) If $x>0$ and $y \in \mathbb{R}$ then show that there exists a natural number n such that $nx>y$.

(e) Prove that a monotonic increasing bounded above sequence converges to its least upper bound.

(f) Prove that the limit of a sequence is unique.

(g) State Cauchy's General Principle of convergence of a series. Using this show that the series $\Sigma \frac{1}{n}$ does not converge.

N.B:- Write your name, class roll no & examination roll no in your answer script.

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