HOME ASSIGNMENT

B.Sc. 4th Semester

Sub: Mathematics (Major)

Paper: 4.1 (Real Analysis)

Total Marks: 50

Instructions:

- Date of submission: On or before 8th August, 2020.
- Write your name, class roll no. & examination roll no. in your answerscript.
- Upload your answerscript only in PDF format on the portal.

1. Answer the following questions: $1 \times 7 = 7$

(a) Write true or false: A finite set has no limit point.

(b) Define a Cauchy sequence.

(c) Find the infimum of the set $\{ \frac{(-1)^n}{n}, n \in \mathbb{N} \}$

(d) The positive term series $\sum \frac{1}{n^p}$ is convergent if and only if (i) $p>1$, (ii) $p=0$, (iii) $0<p<1$, (iv) $p \leq 1$ (Choose the correct answer)

(e) If $a_n = (-1)^n$ and $b_n = (-1)^{n+1}$, then the sequence $\{a_n b_n\}$ is always convergent. (Write true or false)

(f) Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+ \cdots\cdots$ is not convergent.

(g) If $\{x_n\}$ is a sequence, where $x_n = k$ ($k \in \mathbb{R}$) is constant, then $\lim x_n = k$ (Write true or false)

2. Answer the following questions: $2 \times 5 = 10$

(a) Any open interval $I = (a, b)$ is an open set. Why?

(b) Illustrate with an example that bounded sequence is not convergent.

(c) Show that the sequence $\left\{\frac{2n-7}{3n+2}\right\}$ is bounded

(d) If S_1, S_2 are subsets of $\mathbb{R}$, then show that

$$(S_1 \cap S_2)' \subseteq S_1' \cap S_2'$$

(e) Show that the sequence $\{x_n\}$ defined by recursion formula $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2x_n}$ is monotonically increasing.

3. Answer the following questions : (any two) $4 \times 2 = 8$

(a) Prove that $\lim_{n \to \infty} n^{1/n} = 1$

(b) Prove that the arbitrary intersection of closed sets is closed.

(c) Applying Sandwich theorem, show that the sequence $\{x_n\}$, where $x_n = \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \frac{1}{\sqrt{n^2+3}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right]$ converges to 1.

4. Answer the following questions : (any five) $5 \times 5 = 25$

(a) If a and b be any two positive real numbers, then show that there exists a positive integer n such that $na > b$.

(b) Test for convergence of the following series whose nth term is given by $\frac{1.3.5 \cdots\cdots (4n-3)}{2.4.6 \cdots\cdots (4n-2)} \cdot \frac{x^{2n}}{4n}$

(c) State and prove Sandwich theorem for sequence of real numbers.

(d) State Cauchy's first theorem on limit of a sequence. Applying this theorem, prove that

$$\lim_{n \to \infty} \left[\frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots\cdots + \frac{1}{(2n)^2}\right] = 0$$

(e) Show that the sequence $\{x_n\}$, where $x_n = \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots + \frac{1}{\lfloor n}$ is convergent.

(f) State and prove Bolzano-Weierstrass theorem for sets.

(g) If $\{a_n\}$ be a sequence such that $\lim \frac{a_{n+1}}{a_n} = l$ where $|l| < 1$ then show that $\lim a_n = 0$.

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