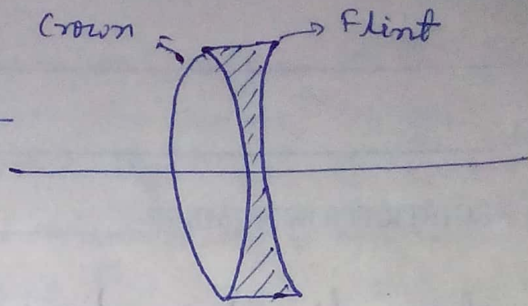
Acromatic lenses:

1. **Condition for achromatism of two lenses placed in contact:**



Let μ_b, μ, μ_r and μ'_b, μ', μ'_r be the refractive indices for blue, yellow and red rays of light of the two materials. f_b, f, f_r and f'_b, f', f'_r are the corresponding ~~forcal~~ focal lengths for the two lenses and ω and ω' are the dispersive powers for crown and flint glass respectively.

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{---(i)}$$

$$\frac{1}{f_b} = (\mu_b-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{---(ii)}$$

$$\frac{1}{f_r} = (\mu_r-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{---(iii)}$$

$$\frac{1}{f'} = (\mu'-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{---(iv)}$$

$$\frac{1}{f'_b} = (\mu'_b-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{---(v)}$$

$$\frac{1}{f'_r} = (\mu'_r-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{---(vi)}$$

From (i) and (iv)

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{(\mu-1)f} \quad \text{---(vii)}$$

$$\left(\frac{1}{R'_1} - \frac{1}{R'_2}\right) = \frac{1}{(\mu'-1)f'} \quad \text{---(viii)}$$

Substituting these values in (ii), (iii), (v) and (vi)

$$\frac{1}{f_b} = \frac{\mu_b-1}{(\mu-1)f}$$

$$\frac{1}{f_r} = \frac{(\mu_r-1)}{(\mu-1)f}$$

$$\frac{1}{f'_b} = \frac{\mu'_b-1}{(\mu'-1)f'}$$

$$\frac{1}{f'_r} = \frac{(\mu'_r-1)}{(\mu'-1)f'}$$

Let F_b and F_r be the focal lengths of the combination for blue and red rays of light. Then

$$\frac{1}{F_b} = \frac{1}{f_b} + \frac{1}{f'_b} = \frac{\mu_b - 1}{(\mu-1)f} + \frac{\mu'_b - 1}{(\mu'-1)f'}$$

$$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f'_r} = \frac{\mu_r - 1}{(\mu-1)f} + \frac{\mu'_r - 1}{(\mu'-1)f'}$$

For the combination to be achromatic the focal lengths F_b and F_r must be equal.

$$F_b = F_r \Rightarrow \frac{1}{F_b} = \frac{1}{F_r}$$

$$\Rightarrow \frac{\mu_b - 1}{(\mu-1)f} + \frac{\mu'_b - 1}{(\mu'-1)f'} = \frac{\mu_r - 1}{(\mu-1)f} + \frac{\mu'_r - 1}{(\mu'-1)f'}$$

$$\Rightarrow \frac{\mu_b - \mu_r}{(\mu-1)f} + \frac{\mu'_b - \mu'_r}{(\mu'-1)f'} = 0$$

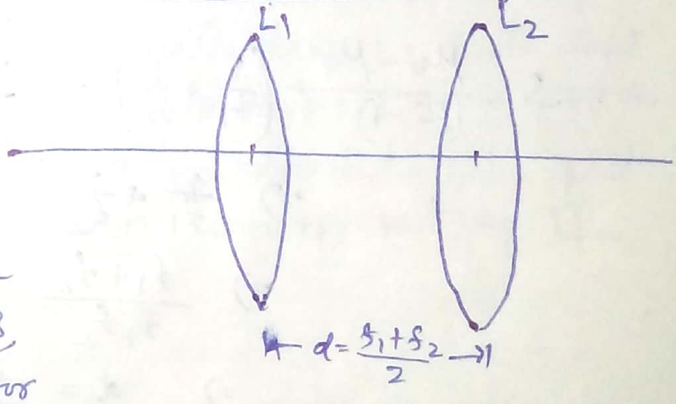
$$\Rightarrow \frac{\omega}{f} + \frac{\omega'}{f'} = 0$$

$$\frac{\omega}{f} = -\frac{\omega'}{f'}$$

$$\Rightarrow f' = -f\frac{\omega'}{\omega}$$

where ω and ω' are the dispersive powers of the two materials, f and f' are the focal lengths of the two lenses.

Condition for achromatism of two thin lenses separated by a finite distance:



Let f_1 and f_2 be the focal lengths of two lenses separated by a distance d. The two lenses are made of the same material and μ, μ_b and μ_r are the refractive indices for the mean rays, blue rays and red rays respectively. For both the lenses f_r, f'_r and f_b, f'_b are the focal lengths of the two lenses for red and blue rays of light.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f'_r} - \frac{d}{f_r f'_r}$$

$$\frac{1}{F_b} = \frac{1}{f_b} + \frac{1}{f'_b} - \frac{d}{f_b f'_b}$$

where F, F_r and F_b are the focal lengths of the combination for the mean rays, red rays and blue rays.

$$\frac{1}{f_r} = \frac{\mu_r - 1}{(\mu - 1)f_1} \; ; \; \frac{1}{f_r'} = \frac{\mu_r - 1}{(\mu - 1)f_2}$$

$$\frac{1}{f_b} = \frac{\mu_b - 1}{(\mu - 1)f_1} \; ; \; \frac{1}{f_b'} = \frac{\mu_b - 1}{(\mu - 1)f_2}$$

$$\frac{1}{F_r} = \frac{\mu_r - 1}{(\mu - 1)f_1} + \frac{\mu_r - 1}{(\mu - 1)f_2} - \frac{(\mu_r - 1)^2}{(\mu - 1)^2} \cdot \frac{d}{f_1 f_2}$$

$$\frac{1}{F_b} = \frac{\mu_b - 1}{(\mu - 1)f_1} + \frac{\mu_b - 1}{(\mu - 1)f_2} - \frac{(\mu_b - 1)^2}{(\mu - 1)^2} \cdot \frac{d}{f_1 f_2}$$

For the combination to be achromatic

$$F_r = F_b$$

$$\Rightarrow \frac{\mu_r - 1}{(\mu - 1)f_1} + \frac{\mu_r - 1}{(\mu - 1)f_2} - \frac{d(\mu_r - 1)^2}{(\mu - 1)^2 f_1 f_2} = \frac{\mu_b - 1}{(\mu - 1)f_1} + \frac{\mu_b - 1}{(\mu - 1)f_2} - \frac{d(\mu_b - 1)^2}{(\mu - 1)^2 f_1 f_2}$$

$$\Rightarrow \frac{(\mu_r - 1)}{(\mu - 1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) - \frac{d(\mu_r - 1)^2}{(\mu - 1)^2 f_1 f_2} = \frac{\mu_b - 1}{(\mu - 1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) - \frac{d(\mu_b - 1)^2}{(\mu - 1)^2 f_1 f_2}$$

$$\Rightarrow \frac{\mu_b - \mu_r}{(\mu - 1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) = \frac{d}{(\mu - 1)^2 f_1 f_2}\left[(\mu_b - 1)^2 - (\mu_r - 1)^2\right]$$

$$= \frac{d}{(\mu - 1)^2 f_1 f_2}\left[(\mu_b - \mu_r)(\mu_b + \mu_r - 2)\right]$$

Taking $\mu_b + \mu_r = 2\mu$

$$= \frac{d(\mu_b - \mu_r)}{(\mu - 1)^2 f_1 f_2} \cdot 2(\mu - 1)$$

$$\therefore \frac{\mu_b - \mu_r}{(\mu - 1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) = \frac{2d(\mu_b - \mu_r)}{(\mu - 1) f_1 f_2}$$

$$\Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = \frac{2d}{f_1 f_2}$$

$$\Rightarrow \frac{f_1 + f_2}{f_1 f_2} = \frac{2d}{f_1 f_2}$$

$$\Rightarrow d = \frac{f_1 + f_2}{2}$$

Q: Two glasses, have dispersive powers in the ratio 2:3. These glasses are to be used in the manufacture of an achromatic objective of focal length 20 cm. What are the focal lengths of the lenses?

Soln: For an achromatic objective

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \; ; \; \Rightarrow \frac{1}{f_2} = -\frac{\omega_1}{\omega_2 f_1}$$

$$\frac{\omega_1}{\omega_2} = \frac{2}{3} \; ; \quad F = 20 \text{ cm}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{20} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_1} - \frac{2}{3f_1} = \frac{1}{3f_1}$$

$f_1 = \frac{20}{3} = 6.67 \text{ cm}$

$\frac{1}{f_2} = -\frac{2}{3} \times \frac{3}{20} = -\frac{1}{10} \Rightarrow f_2 = -10 \text{ cm}$