$$= f_{xx}\, dx^2 + f_{yx} \cdot dy . dx + f_{xy}\, dx \cdot dy + f_{yy}\, dy^2 \quad \text{— (9)}$$

or $$d^2 Z = f_{xx}\, dx^2 + 2 f_{xy}\, dx . dy + f_{yy}\, dy^2 \quad \text{— (10)}$$

$$[\because f_{xy} = f_{yx}]$$

For maximisation, Since $d^2 z < 0$, we can have $d^2 z < 0$, if and only if

$$\boxed{f_{xx} < 0 \;;\; f_{yy} < 0 \;; \text{ and } f_{xx} f_{yy} > f_{yy}^2}$$

The determinant of the above value

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} > 0$$

This above determinant formed by second order partial derivative of Z w.r.t. x and y is called a "Hessian determinant of order 2", and is ~~Sybo~~ Symbolised by $|H_2|$. Thus the necessary and sufficient condition of maximisation of $z = f(x, y)$ are

(1) $f_x = 0$ and $f_y = 0$

(2) $f_{xx} < 0$ (or $|H_1| < 0$) and $|H_2| = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{xy} \end{vmatrix} > 0$

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Optimization with two variables

$$z = f(x, y) \quad \text{———} \quad (1)$$

$$dz = f_x \cdot dx + f_y \cdot dy \quad \text{———} \quad (2)$$

For maximization of $z = f(x, y)$, the first order condition is that

$$dz = f_x \cdot dx + f_y \cdot dy = 0 \quad \text{———} \quad (3)$$

$dz = 0$, if and ~~if~~ only if $f_x = 0$ & $f_y = 0$

Therefore, the first order condition of maximisation is

$$f_x = 0 \text{ and } f_y = 0 \quad \text{———} \quad (4)$$

The second order condition

The differential version of second order condition of maximisation is that $d^2z < 0$.

$$d^2z = d(dz) \quad \text{———} \quad (5)$$

$$= \frac{\partial}{\partial x}(dz) \cdot dx + \frac{\partial}{\partial y}(dz) \cdot dy \quad \text{———} \quad (6)$$

$$= \frac{\partial}{\partial x}\left[f_x \cdot dx + f_y \cdot dy\right] dx + \frac{\partial}{\partial y}\left[f_x \cdot dx + f_y \cdot dy\right] dy \quad \text{———} \quad (7)$$

$$= \left[f_{xx}\, dx + f_{yx}\, dy\right] dx + \left[f_{xy}\, dx + f_{yy}\, dy\right] dy \longrightarrow 8$$

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