# CIRCLE

① The equation of a circle whose centre is at the origin and radius is 'a' is

$$\boxed{x^2 + y^2 = a^2}$$

② The equation of circle whose centre is at $(h, k)$ & radius is 'a' is

$$\boxed{(x-h)^2 + (y-k)^2 = a^2}$$

③ The equation of tangent to the circle $x^2+y^2 = a^2$ at the point $(x_1, y_1)$ is

$$\boxed{x x_1 + y y_1 = a^2.}$$

④ The equation of normal to the circle $x^2+y^2 = a^2$ at the point $(x_1, y_1)$ is

$$\boxed{\frac{x}{x_1} = \frac{y}{y_1}}$$

⑤ The general equation of circle is

$$\boxed{x^2 + y^2 + 2gx + 2fy + c = 0.}$$

Centre is $(-g, -f)$ and radius is $\sqrt{g^2+f^2-c}$.

⑥ The equation of tangent to the circle $x^2+y^2+2gx+2fy+c=0$ at $(x_1, y_1)$ is

$$\boxed{xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0}$$

⑦ The line $y=mx+c$ will be a tangent to the circle $x^2+y^2=a^2$ if

$$c^2=a^2(1+m^2)$$

or, $\boxed{c=\pm a\sqrt{1+m^2}}$

⑧ Any tangent to the circle $x^2+y^2=a^2$ can be taken as $\boxed{y=mx+a\sqrt{1+m^2}}$

Its point of contact is $\left(-\dfrac{am}{\sqrt{1+m^2}}, \dfrac{a}{\sqrt{1+m^2}}\right)$

⑨ The equation of polar of the point $(x_1, y_1)$ w.r.t. circle $x^2+y^2=a^2$ is $\boxed{xx_1+yy_1=a^2}$

⑩ The equation of polar of $(x_1, y_1)$ w.r. to circle $x^2+y^2+2gx+2fy+c=0$ is

$$\boxed{xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0}$$

## Parametric form of Equations of a Circle

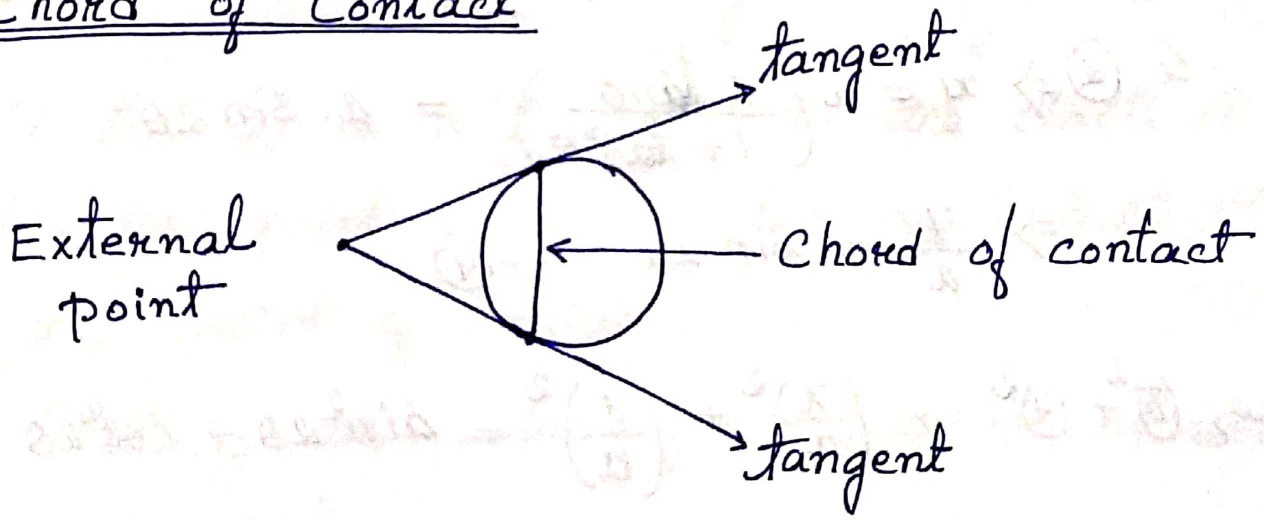
# $\boxed{x = a\cos\theta}$ and $\boxed{y = a\sin\theta}$

are called parametric equations of the circle $x^2+y^2=a^2$.

# $\boxed{x = h + a\cos\theta}$ and $\boxed{y = k + a\sin\theta}$

are called parametric equations of the circle $(x-h)^2+(y-k)^2 = a^2$.

## Chord of Contact




The chord joining the points of contact of tangents to a circle from an external point is called chord of contact.

# The eq$^n$ of chord of contanct of tangent from $(x_1, y_1)$ to the circle $x^2+y^2=a^2$ is

$$\boxed{xx_1 + yy_1 = a^2}$$

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Ex: 1 Show that the parametric equations $x = a\left(\frac{1-t^2}{1+t^2}\right)$, $y = a\left(\frac{2t}{1+t^2}\right)$ represent a circle. Find its centre & radius.

Sol$^n$ Given, $x = a\left(\frac{1-t^2}{1+t^2}\right)$ — (1), $y = a\left(\frac{2t}{1+t^2}\right)$ — (2)

Let $t = \tan\theta$

Then (1) $\Rightarrow$ $x = a\left(\frac{1-t^2}{1+t^2}\right) = a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$$\Rightarrow x = a.\cos 2\theta$$

$$\Rightarrow \frac{x}{a} = \cos 2\theta \quad \text{— (3)}$$

& (2) $\Rightarrow$ $y = a\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = a.\sin 2\theta$

$$\Rightarrow \frac{y}{a} = \sin 2\theta \quad \text{— (4)}$$

Now, $(3)^2 + (4)^2 \Rightarrow \left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = \sin^2 2\theta + \cos^2 2\theta$

$$\Rightarrow \frac{x^2+y^2}{a^2} = 1$$

$\Rightarrow x^2 + y^2 = a^2$, which is a circle with centre at the origin and radius 'a'.

Ex: ② If the line $3x - 2y = 18$ is a tangent to the circle $x^2 + y^2 + 5x + 6y - 14 = 0$, find the point of contact.

Sol:ⁿ Given, eq.ⁿ of the circle is

$$x^2 + y^2 + 5x + 6y - 14 = 0 \quad \text{--- ①}$$

& eq.ⁿ of the line is $3x - 2y = 18$.

$$\Rightarrow 3x - 18 = 2y$$

$$\Rightarrow \frac{3x-18}{2} = y$$

$$\Rightarrow y = \frac{3x-18}{2} \quad \text{--- ②}$$

∵ Line ② is a tangent to the circle ①.

So, the point of intersection of ① & ② is the point of contact.

Putting the value $y = \frac{3x-18}{2}$ in eq.ⁿ ①, we get

$$x^2 + \left(\frac{3x-18}{2}\right)^2 + 5x + 6\left(\frac{3x-18}{2}\right) - 14 = 0$$

$$\Rightarrow x^2 + \frac{9x^2 - 2.3x.18 + (18)^2}{4} + 5x + 3(3x-18) - 14 = 0$$

$$\Rightarrow x^2 + \frac{9x^2 - 108x + 324}{4} + 5x + 9x - 54 - 14 = 0$$

$$\Rightarrow 4x^2 + 9x^2 - 108x + 324 + 20x + 36x - 216 - 56 = 0$$

$$\Rightarrow 13x^2 - 52x + 52 = 0$$

$$\Rightarrow x^2 - 4x + 4 = 0$$

$$\Rightarrow (x-2)^2 = 0$$

$$\Rightarrow x = 2.$$

Putting this value in ② we get

$$y = \frac{3\times 2 - 18}{2} = \frac{6-18}{2} = \frac{-12}{2} = -6$$

$\therefore$ The point of contact is $(2, -6)$.

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③ Find the equation of the pair of tangents from a given point $(\alpha, \beta)$ to the circle $x^2 + y^2 = a^2$.

Solⁿ: The eqⁿ of any tangent to the circle $x^2 + y^2 = a^2$ is $y = mx + a\sqrt{1+m^2}$ — ①

$\because$ ① passes through the point $(\alpha, \beta)$

So, ~~y~~ $\beta = m\alpha + a\sqrt{1+m^2}$ — ②

Now, ① − ② $\Rightarrow$ $y-\beta = m(x-\alpha)$

$$\Rightarrow \quad m = \frac{y-\beta}{x-\alpha}$$

Putting this value in ①, we get

$$y = \left(\frac{y-\beta}{x-\alpha}\right)x + a\sqrt{1+\left(\frac{y-\beta}{x-\alpha}\right)^2}$$

$$\Rightarrow \left\{y - \frac{(y-\beta)x}{x-\alpha}\right\}^2 = \left\{a\sqrt{1+\left(\frac{y-\beta}{x-\alpha}\right)^2}\right\}^2$$

$$\Rightarrow \left(\frac{xy-\alpha y - xy + \beta x}{x-\alpha}\right)^2 = a^2\left[\frac{(x-\alpha)^2 + (y-\beta)^2}{(x-\alpha)^2}\right]$$

$$\Rightarrow (\beta x - \alpha y)^2 = a^2\left[x^2 - 2x\alpha + \alpha^2 + y^2 - 2y\beta + \beta^2\right]$$

$$\Rightarrow \beta^2x^2 + \alpha^2y^2 - 2\beta\alpha xy = a^2x^2 - 2a^2\alpha x + a^2\alpha^2 + a^2y^2 - 2a^2y\beta + a^2\beta^2$$

$$\Rightarrow \beta^2x^2 + \alpha^2y^2 - a^2x^2 - a^2\alpha^2 - a^2y^2 - a^2\beta^2 = 2\alpha\beta xy - 2a^2\alpha x - 2a^2y\beta$$

$$\Rightarrow x^2(\beta^2 - a^2) + y^2(\alpha^2 - a^2) - a^2(\alpha^2 + \beta^2) = 2\alpha\beta xy - 2a^2\alpha x - 2a^2y\beta$$

Adding $\alpha^2x^2 + \beta^2y^2 + a^4$ both sides, we get

$$\Rightarrow x^2(\alpha^2+\beta^2-a^2) + y^2(\alpha^2+\beta^2-a^2) - a^2(\alpha^2+\beta^2-a^2)$$
$$= (\alpha x)^2 + (\beta y)^2 + (-a^2)^2 + 2\alpha x\beta y + 2\beta y(-a^2) + 2(-a^2)\alpha x$$

$$\Rightarrow (x^2+y^2-a^2)(\alpha^2+\beta^2-a^2) = (\alpha x+\beta y-a^2)^2$$

$$\Rightarrow \quad S S_1 = T^2$$

where $S = x^2+y^2-a^2$

$S_1 = \alpha^2+\beta^2-a^2$

& $T = \alpha x+\beta y-a^2$.

This is the required eqⁿ of the pair of tangents from a given point $(\alpha,\beta)$ to the circle $x^2+y^2=a^2$.

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④ Prove that the polars of $(1,-2)$ w.r.t. the circle $x^2+y^2+6y+5=0$ & $x^2+y^2+2x+8y+5=0$ are the same.

Solⁿ: We know, the equation of polar of $(x_1,y_1)$ w.r.t. circle $x^2+y^2+2gx+2fy+c=0$ is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

∴ The eqⁿ of the polar of $(1,-2)$ w.r.t. circle $x^2+y^2+6y+5=0$

or, $x^2+y^2+2\times 3y+5=0$. is

—— ①

$x \times 1 + y \times (-2) + 0 + 3(y \quad ) + 5 = 0$

$\Rightarrow x - 2y + 3y - 6 + 5 = 0$

$\Rightarrow x + y - 1 = 0$ ——— (2)

Again, the eqn of polar of $(1, -2)$ w.r.t. circle $x^2 + y^2 + 2x + 8y + 5 = 0$

i.e. $x^2 + y^2 + 2 \times 1\, x + 2 \times 4\, y + 5 = 0$ is ——— (3)

$x \times 1 + y \times (-2) + 1(x + 1) + 4(y - 2) + 5 = 0$

$\Rightarrow x - 2y + x + 1 + 4y - 8 + 5 = 0$

$\Rightarrow 2x + 2y - 2 = 0$

$\Rightarrow x + y - 1 = 0$. ——— (4)

So Eqns (2) & (4) are same.

$\therefore$ The polars of $(1, -2)$ w.r.t. circle (1) & (3) are same.

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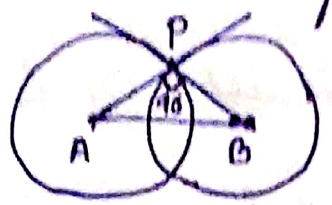
## Orthogonal Circles :

Two circles are said to be orthogonal if the tangents at their common points are at right angeles



Remember: If $x^2+y^2+2g_1x+2f_1y+c_1=0$ &

$$x^2+y^2+2g_2x+2f_2y+c_2=0$$

are two circles. Then the condition for orthogonality of the two circles is

$$\boxed{2g_1g_2+2f_1f_2=c_1+c_2}$$

Ex:① Prove that the circles $x^2+y^2+2x+4y-20=0$ & $x^2+y^2+6x-8y+10=0$ cut orthogonally.

Sol.ⁿ Given, circle is $x^2+y^2+2x+4y-20=0$. —①

Comparing with $x^2+y^2+2g_1x+2f_1y+c_1=0$,

we get $g_1=1$, $f_1=2$, $c_1=-20$

Again, circle given is $x^2+y^2+6x-8y+10=0$ —②

Comparing with $x^2+y^2+2g_2x+2f_2y+c_2=0$,

we get $g_2=3$, $f_2=-4$, $c_2=10$

Now, $2g_1g_2 + 2f_1f_2 = 2\times1\times3 + 2\times2\times(-4)$
$= 6 - 16$
$= -10$

& $c_1 + c_2 = -20 + 10 = -10$

$\therefore 2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

which is the condition of orthogonality.

Hence the circles ① & ② are orthogonal.

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(2) Determine the value of $k$, so that circles $x^2+y^2+5x+3y+7=0$ and $x^2+y^2-8x+6y+k=0$ are orthogonal.

Soln Given, Circle is $x^2+y^2+5x+3y+7=0$ — ①

Comparing with $x^2+y^2+2g_1x+2f_1y+c_1=0$,

we get $g_1 = \frac{5}{2}$ ; $f_1 = \frac{3}{2}$, $c_1 = 7$

Another circle is $x^2+y^2-8x+6y+k=0$ — ②

Comparing with $x^2+y^2+2g_2x+2f_2y+c_2=0$,

we get $g_2 = -4$, $f_2 = 3$, $c_2 = k$

$\because$ The circles are orthogonal

$\therefore 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow 2\times\frac{5}{2}\times(-4) + 2\times\frac{3}{2}\times3 = 7 + k$

$\Rightarrow -20 + 9 - 7 = k$

$\Rightarrow k = -11 - 7 = -18$ //

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