Frame of reference: An inertial frame of reference is one in which Newton's first law of motion holds. In such a frame, an object at rest remains at rest and an object in motion continues to moves at constant velocity (constant speed and direction) if no force acts on it. Any frame of reference that moves at constant velocity relative to an inertial frame itself an inertial frame.

Postulates of special Relativity:

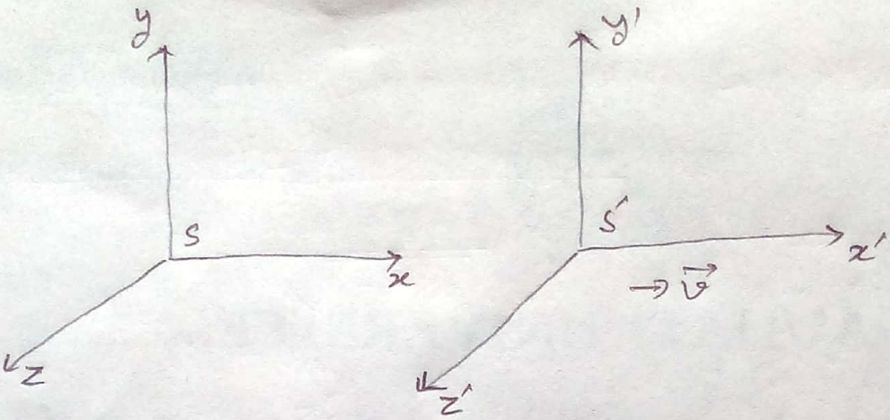
Two postulates underline special relativity. The principle of relativity states:

The law of physics are the same in all inertial frames of reference.

The speed of light in free space has the same value in all inertial frames of reference. The speed of light is 2.998×10^8 m/s.

Galilean Transformation: Consider two reference frame s and s', origin of which are coincide when the time in the clocks started. The measurement in the x-direction made in s will be greater than those made in s' by the amount vt which is the distance s' has moved in the x-direction.

$$x' = x - vt \qquad \text{——— (1)}$$



There is no relative motion in the y and z-directions

$$y' = y \quad \text{——— (2)}$$
$$z' = z \quad \text{——— (3)}$$
$$t' = t \quad \text{——— (4)}$$

This set of equation is known as the Galilean tranformation

Lorentz transformation: A reasonable guess about the nature of the correct relationship between x and x' is

$$x' = \kappa(x - vt) \quad \text{——— (1)}$$

Here κ is a factor that doesnot depend upon either x or t but may be a function of v.

It is linear in x and x', so that a single event in S frame corresponds to a single event in S' frame.

$$x = \kappa(x' + vt') \quad \text{——— (2)}$$

The factor κ must be the same in both frames of reference since there is no difference between S and S' other than the sign of v. There is nothing to indicate that there might be differences between the corresponding coordinates y, y' and z, z' which are perpendicular to the direction of v.

$$y' = y \quad \text{——— (3)}$$
$$z' = z \quad \text{——— (4)}$$

The time coordinates t and t' however are not equal.

$$x = \kappa\{\kappa(x - vt)\} + \kappa v t'$$
$$= \kappa^2 (x - vt) + \kappa v t'$$
$$\kappa v t' = x + \kappa^2 vt - \kappa^2 x = \kappa^2 vt + (1 - \kappa^2)x$$
$$t' = \kappa t + \left(\frac{1 - \kappa^2}{\kappa v}\right)x \quad \text{——— (5)}$$

The second postulates of relativity gives us a way to evaluate κ. At the instant $t = 0$, the origins of the two frames of reference S and S' are in the same place.

Suppose that a flare is set off at the common origin of s and s' at $t = t' = 0$, and the observers in each system measure the speed with which the flare's light spreads out. Both observers must find the same speed 'c' which means that in the S-frame

$$x = ct \quad \text{—— (6)}$$

and in the S'-frame $\quad x' = ct' \quad$ —— (7)

Substituting for x' and t' in eqn (7)

$$K(x - vt) = c\left\{Kt + \left(\frac{1-K^2}{Kv}\right)x\right\}$$

$$Kx - Kvt = cKt + \left(\frac{1-K^2}{Kv}\right)cx$$

$$\left\{K - \left(\frac{1-K^2}{Kv}\right)c\right\}x = cKt + vKt$$

$$\Rightarrow x = \frac{cKt + vKt}{K - \left(\frac{1-K^2}{Kv}\right)c}$$

$$\Rightarrow x = ct\left[\frac{K + \frac{v}{c}K}{K - \left(\frac{1-K^2}{Kv}\right)c}\right] = ct\left[\frac{1 + v/c}{1 - \left(\frac{1}{K^2} - 1\right)c/v}\right]$$

This expression for x will be the same as that given by $x = ct$ provided that the quantity in the brackets equals 1.

Therefore

$$\frac{1 + v/c}{1 - \left(\frac{1}{K^2} - 1\right)c/v} = 1$$

$$\Rightarrow 1 + v/c = 1 - \left(\frac{1}{K^2} - 1\right)c/v$$

$$\Rightarrow v/c = \left(\frac{1}{K^2} - 1\right)c/v$$

$$\Rightarrow \frac{1}{K^2} = 1 - v^2/c^2$$

$$\Rightarrow K^2 = \frac{1}{1 - v^2/c^2}$$

$$\Rightarrow K = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \text{—— (8)}$$

Lorentz transformation $\quad x' = \dfrac{x - vt}{\sqrt{1 - v^2/c^2}}$ —— (9)

$$y' = y \quad \text{—— (10)}$$

$$z' = z \quad \text{—— (11)}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}} \quad \text{—— (12)}$$

These equations comprise the Lorentz transformation.

Inverse Lorentz transformation:

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Velocity addition:

Suppose something is moving relative to both s and s'. An observer in s measures its three velocity components to be $V_x = \frac{dx}{dt}$, $V_y = \frac{dy}{dt}$, $V_z = \frac{dz}{dt}$

while to an observer in s' they are

$$V_x' = \frac{dx'}{dt'}, \quad V_y' = \frac{dy'}{dt'}, \quad V_z' = \frac{dz'}{dt'}$$

By differentiating the inverse Lorentz transformation equations for x, y, z and t, we obtain

$$dx = \frac{dx' + v\,dt'}{\sqrt{1 - v^2/c^2}}; \quad dy = dy', \quad dz = dz', \quad dt = \frac{dt' + \frac{v\,dz'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$V_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v\,dx'}{c^2}} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$

$$V_x = \frac{V_x' + v}{1 + \frac{v V_x'}{c^2}}$$

$$V_y = \frac{V_y' \sqrt{1 - v^2/c^2}}{1 + \frac{v V_x'}{c^2}}; \quad V_z = \frac{V_z' \sqrt{1 - v^2/c^2}}{1 + \frac{v V_x'}{c^2}}$$

If $V_x' = c$, if light is emitted in the moving frame s' in its direction of motion relative to S, an observer in frame s will measure the speed

$$V_x = \frac{V_x' + v}{1 + \frac{v V_x'}{c^2}} = \frac{c + v}{1 + \frac{vc}{c^2}} = \frac{c(c+v)}{c+v} = c$$

Thus observers in the car and on the road both find the same value for the speed of light, as they must.