Time dilation: Measurements of time intervals are affected by relative motion between an observer and what is observed. If someone in a moving spacecraft finds that the time interval between two events in the spacecraft is t_0, we on the ground would find that the same interval has the longer duration t. The quantity t_0, which is determined by events that occur at the same place in an observer's frame of reference, is called the proper time of the interval between the events. When witnessed from the ground, the events that mark the begining and end of time interval occur at different places and in consequence the duration of the interval appears longer than the proper time. This effect is called time dilation.

Let us consider two clocks, in each clock a pulse of light is reflected back and forth between two mirrors L_0 apart. Whenever light strikes the lower mirror, an electric signal is produced. ~~that~~ that marks the recording tape. Each mark corresponds to the tick of an ordinary clock. One clock is at rest in a laboratory on the ground and the other is in a spacecraft that moves at the speed v relative to the ground.

The time interval between ticks is the proper time t_0 in the laboratory clock and the time needed for the light pulse to travel between the mirrors at the speed of light c is $t_0/2$. Hence $t_0/2 = L_0/c \Rightarrow t_0 = \frac{2L_0}{c}$

The moving clock with its mirrors perpendicular to the direction of motion relative to the ground. The time interval between the Ticks is 't'. Because the clock is moving, the light pulse as seen from the ground follows a zigzag path. On this way from the lower mirror to the upper one in the time $t/2$, the pulse travels a horizontal distance of $v(t/2)$ and a total distance of $c(t/2)$. Since L_0 is the vertical distance between the mirrors.

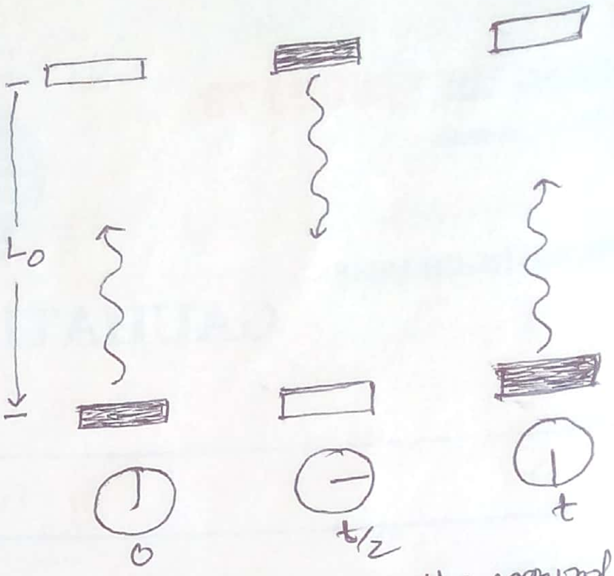

Fig: clock on the ground

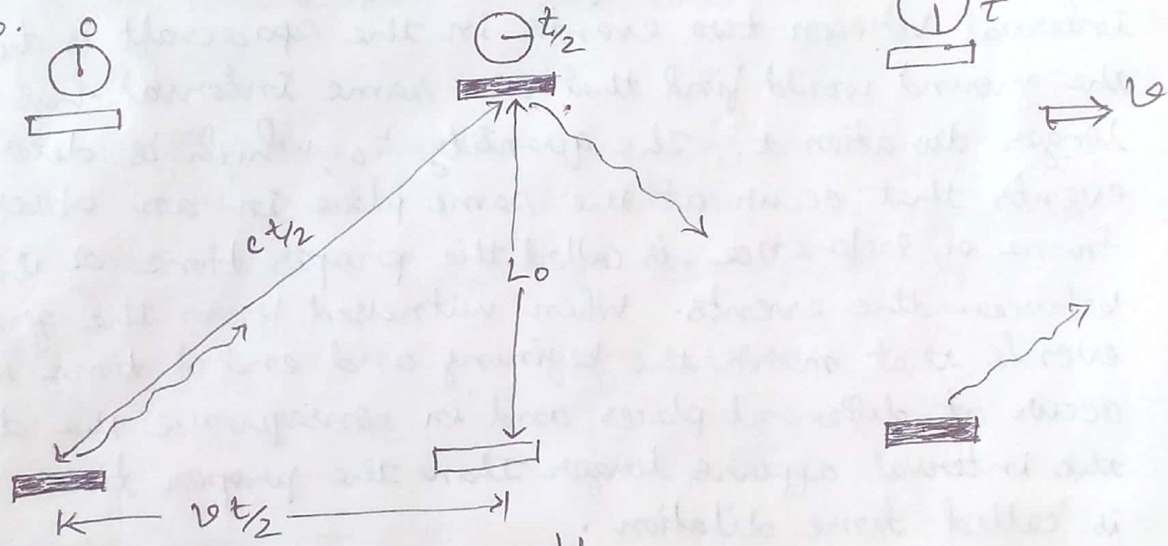

Fig: clock in space craft

$$\left(\frac{ct}{2}\right)^2 = L_0^2 + \left(\frac{vt}{2}\right)^2$$

$$\frac{t^2}{4}(c^2 - v^2) = L_0^2$$

$$t^2 = \frac{4L_0^2}{c^2 - v^2} = \frac{(2L_0)^2}{c^2(1 - v^2/c^2)}$$

$$t = \frac{2L_0/c}{\sqrt{1 - v^2/c^2}}$$

But $2L_0/c$ is the time interval t_0 between ticks on the clock on the ground.

Time dilation $\boxed{t = \frac{t_0}{\sqrt{1 - v^2/c^2}}}$

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Q: A spacecraft is moving relative to earth. An observer on the earth finds that, between 1 p.m. and 2 p.m. according to her clock, 3601 sec elapse on the spacecraft's clock. What is the spacecraft's speed relative to the earth?

Solution: Here $t_0 = 3600$ sec is the proper time interval on the earth and $t = 3601$ sec is the time interval in the ~~earth~~ moving frame as measured from the earth.

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$1 - v^2/c^2 = \left(\frac{t_0}{t}\right)^2$$

$$v = c\sqrt{1 - \left(\frac{t_0}{t}\right)^2} = (3\times10^8 \, m/s)\cdot\sqrt{1 - \left(\frac{3600\,s}{3601\,s}\right)^2}$$

$$v = 7.1\times10^6 \, m/s$$

Length contraction:

Measurements of lengths as well as of time intervals are affected by relative motion. The length L of an object in motion with respect to an observer always appears to the observer to be shorter than its length L_0 when it is at rest with respect to him. This contraction occurs only in the direction of the relative motion. The length L_0 of an object in its rest frame is called its proper length.

Let us consider what happens to unstable particles called muons that are created at high altitudes by fast cosmic-ray particles (largely protons) from space when they collide with atomic nuclei in the earths atmosphere. A muon has a mass 207 times that of the electron and has a charge of either +e or −e; it decays into an electron or a positron after an average life time of 2.2 μs (2.2×10^{-6} s)

Cosmic-ray muons have speeds of about 2.994×10^8 m/s (0.998c) and reach sea level in profusion - one of them passes through each square centimeter of the earths surface on the average slightly more often than once a minute. But in $t_0 = 2.2$ μs their average life time, muons can travel a distance of only

$$vt_0 = (2.994 \times 10^8 \, m/s)(2.2 \times 10^{-6} s) = 6.6 \times 10^2 m = 0.66 \, Km$$

before decaying, whereas they are actually created at altitudes of 6 Km or more.

To resolve the paradox, we note that the muon lifetime of $t_0 = 2.2 \, \mu s$ is what an observer at rest with respect to a muon would find. Because the muons are hurtling toward us at the considerable speed of $0.998c$ their lifetimes are extended in our frame of reference by time dilation

$$t = \frac{t_0}{\sqrt{1-v^2/c^2}} = \frac{2.2 \times 10^{-6} s}{\sqrt{1 - \frac{(0.998c)^2}{c^2}}} = 34.8 \times 10^{-6} s = 34.8 \, \mu s$$

The moving muons have lifetimes almost 16 times longer than those at rest. In a time interval of $34.8 \, \mu s$, a muon whose speed is $0.998c$ can cover the distance

$$vt = (2.994 \times 10^8 \, m/s)(34.8 \times 10^{-6} sec)$$
$$= 1.04 \times 10^4 m = 10.4 \, Km$$

Although its lifetime is only $t_0 = 2.2 \, \mu s$ in its own frame of reference, a muon can reach the ground from altitudes are measured, the muon lifetime is $t = 34.8 \, \mu s$.

The only way to account for the arrival of the muon at ground level is if the distance it travels, from the point of view of an observer in the moving frame, is shortened by virtue of its motion. The principle of relativity tells us the extent of the shortening - it must be ~~the~~ by the same factor of $\sqrt{1-v^2/c^2}$ that the muon lifetime is extended from the point of view of a stationary observer.

We therefore conclude that an altitude we on the ground find to be h_0 must appear in the muons frame of reference as the lower altitude

$$h = h_0 \sqrt{1 - v^2/c^2}$$

$h_0 = 10.4$ Km ; $h = (10.4\text{ km})\sqrt{1 - \frac{(0.998c)^2}{c^2}} = 0.66$ Km

The ~~de~~ relativistic shortening of distances is an example of the general contraction of lengths in the direction of motion

Length contraction $\boxed{L = L_0\sqrt{1 - v^2/c^2}}$

Twin Paradox: This paradox involves two identical clocks, one of which remains on the earth while the other is taken on a voyage into space at the speed v and eventually is brought back. It is customary to replace the clocks with the pair of twins Dick and Jane, a substitution that is perfectly acceptable because the process of life - heartbeats, respiration and so on.

Dick is 20 year old when he takes off on a space voyage at a speed of $0.80c$ to a star 20 light years away. To Jane who stays behind, the pace of Dick's life is slower than her by a factor of

$$\sqrt{1 - v^2/c^2} = \sqrt{1 - \frac{(0.80c)^2}{c^2}} = 0.60 = 60\%$$

To Jane, Dicks heart beats only 3 times for every 5 beats of her heart; Dick takes only 3 breaths for every 5 of hers; Dicks thinks only 3 thoughts for every 5 of hers. Finally Dick returns after 50 years have gone by according to Jane's calander, but to Dick the trip has taken only 30 years. Dick is Therefore 50 years old whereas Jane, the twin who stayed home is ~~80~~ 70 years old.

where is the paradox? If we consider the situation from the point of view of Dick in the spacecraft, Jane on the earth is in motion relative to him at a speed of 0.80c. Should not Jane then be 50 years old when the spacecraft returns, while Dick is then 70 — the precise opposite of what was concluded above?

But the two situations are not equivalent. Dick changed from one inertial frame to a different one when he started out, when he reversed direction to head home, and when he landed on the earth. Jane however remained in the same inertial frame during Dick's whole voyage. The time dilation formula applies to Jane's observations of Dick, but not ~~two~~ to Dick's observations of her.

To look at Dick's voyage from his perspective, we must take into account that the distance L he covers is shortened to

$$L = L_0\sqrt{1-v^2/c^2} = 20\sqrt{1-\frac{(0.80c)^2}{c^2}} = 12 \text{ light years}$$

To Dick, time goes by at the usual rate, but his voyage to the star has taken $L/v = 15$ years and his return voyage another 15 years for a total of 30 years. Dick's life span has not been extended to him, because regardless of Jane's 50 years wait, he has spent only 30 years on the roundtrip.

The nonsymmetric aging of the twins has been verified by experiments in which accurate clocks were taken on an airplane trip round the world and then compared with identical clocks that had been left behind. An observer who departs from an inertial system and then returns after moving relative to that system will always find his or her clocks slow compared with clocks that stayed in the system.