B.Sc 2nd Semester (CBCS)

Physics (Hon.)

Unit : IV

Electrical Circuits

Kirchhoff's law :

Kirchhoff developed two fundamental laws for analysing complicated electrical network consisting of many sources, junctions, branches. These two laws are —

a. Kirchoff's current law (KCL)

b. Kirchoff's voltage law (KVL)

1. Kirchoff's current law (KCL) :

It states that at any instant of time algebraic sum of currents flowing to a junction point of any network is zero. That is —

$$\sum_i I_n = 0$$

We can easily understand from the example shown in fig 1. In fig1. currents I_1, I_3, I_4 are taken positive and currents I_2 and I_5 are negative. Therefore, according to KCL the algebraic sum of currents are as —

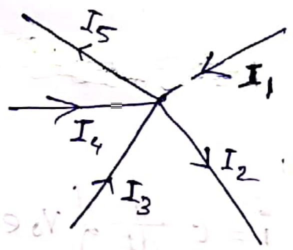


Fig:1 (KCL)

$$I_1 - I_2 + I_3 + I_4 - I_5 = 0$$

2. Kirchoff's voltage law:

It states that at any instant of time the algebraic sum of the voltage drops around a closed path in a network is zero. That is —

$$\sum_i V_n = 0$$

In fig2:-

Here, $IR_1 + IR_2 + IR_3 = V$

or $V_1 + V_2 + V_3 = V$

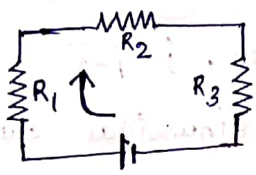
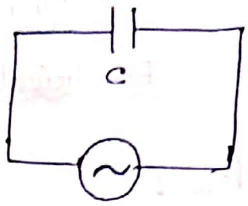


Fig2: (KVL)

Reactance and impedance :

Let us consider a circuit as shown in fig3. The instantaneous current through the capacitor is i at any instant of time t.



$V(t) = V_0 \sin(\omega t + \theta)$

fig:3 AC supply connected to capacitor.

The current i supplied by the ac source connected to a capacitor is given by —

$$i = C\frac{dv}{dt}$$

$$= C\frac{d}{dt}\{V_0 \sin(\omega t + \theta)\}$$

$$= V_0 \omega C \sin(\omega t + \theta) \longrightarrow (1)$$

We can write,

$$\sin(\omega t + \theta) = e^{j(\omega t + \theta)} \quad [\text{In complex notation}]$$

$$\therefore i = C\frac{d}{dt}\{V_0 e^{j(\omega t + \theta)}\}$$

$$i = j\omega C V_0 e^{j(\omega t + \theta)} \longrightarrow (2)$$

comparing equ^n (1) and (2) we get — $1/j\omega C$ plays the role of R in this circuit and current leads the voltage by an angle θ.

The reactance of a capacitor C in an ac circuit is defined as —

$$Z_c = 1/j\omega C \longrightarrow (3)$$

where, $j = \sqrt{-1}$ and $\omega (= 2\pi f)$ is the angular frequency of the sinusoidal current.

Similarly, let us consider the circuit in fig:4. If the current through the inductor be —

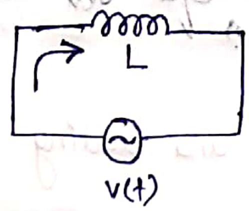


fig:4 : AC supply connected to an indicctor

$$i(t) = I_0 e^{j(\omega t + \theta)},$$

Now,

$$\text{Voltage, } V(t) = L\frac{di}{dt}$$

$$= L\frac{d}{dt}\left\{ I_0 e^{j(\omega t + \theta)} \right\}$$

$$= j\omega L I_0 e^{j(\omega t + \theta)} \longrightarrow (4)$$

Here, $j\omega L$ plays the same role as R and the voltage leads the current by an angle θ.

The reactance of an inductance L in an ac circuit is defined as —

$$Z_L = j\omega L \longrightarrow (5)$$

Again,

Let us consider the current $i(t)$ through a circuit be represented as $I_0 e^{j\omega t}$. The voltage applied to the circuit is given as —

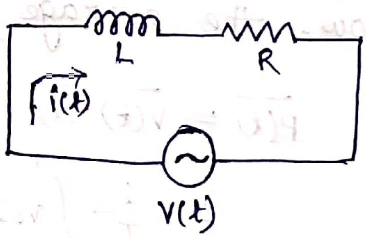


Fig:5 AC connected with series combination of L and R.

$$V(t) = L\frac{di}{dt} + Ri$$

$$= j\omega L I_0 e^{j\omega t} + R I_0 e^{j\omega t} \longrightarrow (6)$$

Let, $R = Z\cos\phi \longrightarrow 6(A)$

and, $j\omega L = Z\sin\phi \longrightarrow 6(B)$

Squaring and adding equation 6(A) and 6(B) we get —

$$R^2 + (j\omega L)^2 = Z^2\cos^2\phi + Z^2\sin^2\phi$$

$$\Rightarrow Z^2 = R^2 + \omega^2 L^2$$

$$\Rightarrow Z = \sqrt{R^2 + \omega^2 L^2} \longrightarrow (7)$$

$\left[\because \cos^2\phi + \sin^2\phi = 1,\ j = \sqrt{-1},\ j^2 = 1 \right]$

and $Z = \sqrt{R^2+\omega^2 L^2}$ is called the impedance of this circuit (fig:5).

Now,

$$\frac{6(B)}{6(A)} \Rightarrow \frac{Z\sin\phi}{Z\cos\phi} = \frac{j\omega L}{R}.$$

$$\Rightarrow \tan\phi = \frac{j\omega L}{R}$$

$$\Rightarrow \phi = \tan^{-1}\frac{j\omega L}{R}.$$

Now, the average power in this circuit is given by —

$$\overline{P(t)} = \overline{V(t)\, i(t)}$$

$$= \frac{1}{T}\int_0^T V_0 I_0 Z e^{j(2\omega t + \phi)}\, dt$$

$$= \frac{V_0 I_0}{2}\cos\phi$$

$$= V_{rms} I_{rms} \cos\phi, \longrightarrow (8)$$

From equ^n (8), Here, $\cos\phi$ is called the power factor.

th

w

s

{

Series resonant circuit :

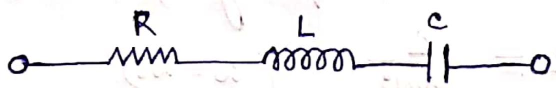


Fig:6 Series LCR circuit.

Let us considere a circuit where resistor (R), inductor (L) and capacitor (C) are connected in series as shown in the fig 6. The series R, L, C circuit shows resonance at some frequency f_0. So this circuit is called series resonant circuit.

From the above fig6: , the total impedance of the series RLC is —

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

and $$\phi = \tan^{-1}\left[\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right]$$

Thus, the current through the circuit i varies with frequency of the ac sinusoidal voltage. For a given value of the peak value of sinusoidal voltage, the variation of the peak current (I) with frequency

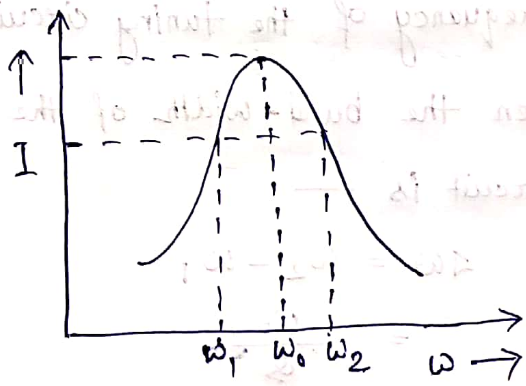


Fig:7 Frequency response of series resonant circuit.

has been shown in fig 8. The current through the circuit will be maximum at a frequency f_0, when

$$\omega_0 L = \frac{1}{\omega_0 C} \quad (\omega = \omega_0)$$

i.e, when $|Z| = R$, the minimum possible magnitude of impedance. So at frequency, $f_0 \left[= 1/2\pi\sqrt{LC}\right]$ the impedance of series R, L, C circuit is minimum, and the current through the circuit is maximum. This phenomenon is called resonance.

The Q-factor of a series resonant is defined as the voltage across the inductance to the voltage across the resistance at resonance. So,

$$Q = \frac{V_L}{V_R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

Large the Q-value more is the responce of the circuit.

Now, if ω_1 and ω_2 are lower and upper half-power frequency of the tuning circuit, then the band-width of the circuit is —

$$\Delta\omega = \omega_2 - \omega_1$$

$$= \frac{\omega_0}{Q}$$

or $$Q = \frac{\omega_0}{\Delta\omega}$$

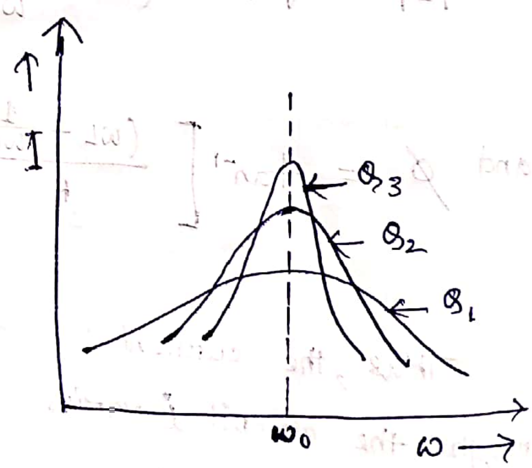


Fig:8 Frequency response of different series resonant circuits with different Q-factor.

So, for lower bandwidth, the Q-value of the circuit is high.