Functions

If two variables x & y are so related that for each value of x there is only one value of y then y is said to be a function of x. Also, y is said to be a single valued function.

Eg: $y = f(x) = 2x + 1$

$\therefore f(5) = (2 \times 5) + 1 = 10 + 1 = 11$

Ex: If $f(x) = x^2 - 3x + 2$, find $f(1)$ and $f(3)$

Soln $f(x) = x^2 - 3x + 2$

$f(1) = (1)^2 - 3 \times 1 + 2 = 1 - 3 + 2 = 3 - 3 = 0$ //

$f(3) = (3)^2 - 3 \times 3 + 2 = 9 - 9 + 2 = 2$ //

Ex: If $f(x) = \frac{ax+b}{bx+a}$, prove that $f(x) f\left(\frac{1}{x}\right) = 1$

Soln $f(x) = \frac{ax+b}{bx+a}$

$$f\left(\frac{1}{x}\right) = \frac{a\frac{1}{x} + b}{b.\frac{1}{x} + a} = \frac{\frac{a+bx}{x}}{\frac{b+ax}{x}} = \frac{a+bx}{b+ax}$$

$$LHS = f(x) \cdot f\left(\frac{1}{x}\right) = \frac{(ax+b)}{(bx+a)} \times \frac{(a+bx)}{(b+ax)} = 1 = RHS.$$

Ex: If $\phi(x) = \log \frac{1-x}{1+x}$ show that $\phi(a) + \phi(b) = \phi\left(\frac{a+b}{1+ab}\right)$

Soln $\phi(x) = \log \frac{1-x}{1+x}$

$\phi(a) = \log \frac{1-a}{1+a}$

$\phi(b) = \log \frac{1-b}{1+b}$

$$RHS = \phi\left(\frac{a+b}{1+ab}\right) = \log \frac{1-\left(\frac{a+b}{1+ab}\right)}{1+\left(\frac{a+b}{1+ab}\right)} = \log \frac{\frac{1+ab-a-b}{(1+ab)}}{\frac{1+ab+a+b}{(1+ab)}}$$

$$= \log \frac{1-a-b+ab}{1+a+b+ab}$$

$$LHS = \phi(a) + \phi(b)$$

$$= \log \frac{1-a}{1+a} + \log \frac{1-b}{1+b}$$

$$= \log \left(\frac{1-a}{1+a}\right) \times \left(\frac{1-b}{1+b}\right)$$

$$= \log \frac{(1-a)(1-b)}{(1+a)(1+b)}$$

$$= \log \frac{1-b-a+ab}{1+b+a+ab}$$

$$= \log \frac{1-a-b+ab}{1+a+b+ab}$$

$$= \phi\left(\frac{a+b}{1+ab}\right)$$

$$= RHS.$$

$$\log m \times n = \log m + \log n$$

Ex: The total cost function $C(x)$ of producing x items is given by $C(x) = 1000 + 5x$, when $0 \leq x \leq 500$

$= 2000 + 4x$, when $500 < x \leq 2000$

Find the cost of producing (i) 430 items (ii) 1200 items.

Soln: (i) For $x = 430$,

$$C(x) = 1000 + 5x$$

$$C(430) = 1000 + 5 \times 430$$

$$= 1000 + 2150$$

$$= 3150 \quad //$$

(ii) For $x = 1200$

$$C(x) = 2000 + 4x$$

$$C(1200) = 2000 + 4 \times 1200$$

$$= 2000 + 4800$$

$$= 6800 \quad //$$

Ex:① If $f(x) = \frac{1-x}{1+x}$, show that $f\left(\frac{1-x}{1+x}\right) = x$

Ex:② If $f(x) = \frac{3x+2}{3x-2}$, prove that $\frac{f(x)+1}{f(x)-1} = \frac{3x}{2}$

Ex:③ If $f(x) = b.\frac{(x-a)}{b-a} + a.\frac{(x-b)}{a-b}$, prove that

$f(a) + f(b) = f(a+b)$.