13/9/19

Equivalence of matrices :

Def^n : If B be a $m \times n$ matrix obtaine from an $m \times n$ matrix A by finite no. of ~~ele~~ E-transformation of A. is called equivalent to B. Symbollically we write $A \sim B$.

The following three properties of the relation $\sim$ in the set of all $m \times n$ matrices are quite obvious.

(I) **Reflexivity :** If A is any $m \times n$ matrix, then $A \sim A$, then A can be obtain from A by E-transformation $R_i \rightarrow k R_i$, where $k = 1$.

(II) **Symmetry :** If $A \sim B$, $B \sim A$. If B can be obtained from A by a finite no. of E-transformation of A, then A can also be obtained from B by a finite no. of E-transformation of B.

(III) **Transitivity :** If $A \sim B$, $B \sim C$, then $A \sim C$. If B can be obtained from A by a series of E-transformation of A & C can be obtained by B by a series of E-transformation of B, then C can be obtained from A by a series of E-transformation of A.

HW Q. Find the rank of a matrix

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

by reducing it to a normal form.

HW Q. Reduce the matrix A to its normal form.

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix} \qquad \rho(A) = 3$$

Practical

Q. Find two non-singular matrices P & Q such that PAQ is in the normal form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$

Also find the rank of A.

Sol^n: we have,

$$A = I_3 A I_3$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \cancel{\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 - R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 \to R_3 - 3R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3 \to C_3 - C_2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 \to R_3 - R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -4 & -1 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_2 \to -\frac{1}{2} R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -4 & -1 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - C_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -4 & -1 & 0 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore PAQ = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

where, $P = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -4 & -1 & 0 \end{bmatrix}$

$$Q = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \rho(A) = 2$$

Q Determine non-singular matrices PAQ such that PAQ is in normal form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}_{3\times 4}$$

Also find the rank of matrix.

Solⁿ: we have,

$$A = I_3 A I_4$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 1 & -4 & 11 & -19 \\ 5 & 1 & 4 & -2 \\ 3 & 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 5R_1$, $R_3 \to R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & -4 & 11 & -19 \\ 0 & 21 & -51 & 93 \\ 0 & 14 & -34 & 62 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 + C_3$, $C_4 \to C_3 + C_4$

$$\begin{bmatrix} 1 & 7 & 11 & -8 \\ 0 & -30 & -51 & 42 \\ 0 & -20 & -34 & 28 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 + C_4 \,,\; C_4 \to C_2 + C_4$

$$\Rightarrow \begin{bmatrix} 1 & 7 & 3 & -1 \\ 0 & -30 & -9 & 12 \\ 0 & -20 & -6 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$C_2 \to C_2 + C_4 \,,\; C_3 \to \frac{1}{3} C_3$

$$\Rightarrow \begin{bmatrix} 1 & 6 & 1 & -1 \\ 0 & -18 & -3 & 12 \\ 0 & -12 & -2 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 3 & 2/3 & 2 \\ 0 & 1 & 1/3 & 1 \end{bmatrix}$$

$C_2 \to \frac{1}{6} C_2 \,,\; C_4 \to C_3 + C_4$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & -3 & 9 \\ 0 & -2 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1 \\ 0 & 1/2 & 2/3 & 8/3 \\ 0 & 1/6 & 1/3 & 4/3 \end{bmatrix}$$

$C_3 \to C_2 - C_3 \,,\; C_4 \to \frac{1}{3} C_4$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & 0 & 3 \\ 0 & -2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & -1/6 & 8/9 \\ 0 & 1/6 & -1/6 & 4/9 \end{bmatrix}$$

$C_4 \to C_2 + C_4$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 2/3 \\ 0 & 1/2 & -1/6 & 25/18 \\ 0 & 1/6 & -1/6 & 11/18 \end{bmatrix}$$

$C_4 \to C_1 - C_4$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & \frac{1}{2} & -\frac{1}{6} & -\frac{25}{18} \\ 0 & \frac{1}{6} & -\frac{1}{6} & -\frac{11}{18} \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & -5 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & \frac{1}{2} & -\frac{1}{6} & -\frac{25}{18} \\ 0 & \frac{1}{6} & -\frac{1}{6} & -\frac{11}{18} \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & -5 \\ 3 & 2 & 10 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & \frac{1}{2} & -\frac{1}{6} & -\frac{25}{18} \\ 0 & \frac{1}{6} & -\frac{1}{6} & -\frac{11}{18} \end{bmatrix}$$

$C_2 \to C_1 - C_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & -1 & -5 \\ 3 & 2 & 10 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & -\frac{1}{2} & -\frac{1}{6} & -\frac{25}{18} \\ 0 & -\frac{1}{6} & -\frac{1}{6} & -\frac{11}{18} \end{bmatrix}$$

$$\therefore PAQ = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

where, $P = \begin{bmatrix} 0 & 0 & 1 \\ -1 & -1 & -5 \\ 3 & 2 & 10 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ 0 & -\frac{1}{2} & -\frac{1}{6} & -\frac{25}{18} \\ 0 & -\frac{1}{6} & -\frac{1}{6} & -\frac{11}{18} \end{bmatrix}$

Q Find the rank of matrix

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

by reducing it to a normal form.

Sol^n: $R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 4 & 2 & 0 & 2 \\ 2 & -2 & 0 & 6 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 4 & 2 & 0 & 2 \\ 1 & -2 & 1 & 2 \\ 2 & -2 & 0 & 6 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$, $\cancel{R_3}$ $R_3 \to R_3 - R_1$, $R_4 \to R_4 - 2R_1$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 6 & 0 & -10 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2 - C_4$

$$\sim \begin{bmatrix} 1 & -4 & 0 & 3 \\ 0 & 16 & 0 & -10 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to \frac{1}{4} C_2$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 4 & 0 & -10 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_1 + C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 4 & 0 & -10 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_3 + C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 4 & 0 & -10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to \frac{1}{4} C_2 \;,\; C_4 \to C_2 + C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to \frac{1}{3} C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_1 - C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 - 2C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus, the matrix A is equivalent to the matrix $\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$

Hence, the rank is 3.

Q ~~Find the rank of matrix~~

Q Reduce the matrix A to its normal form

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

Sol^n: $C_2 \to \frac{1}{2} C_2$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 4 \\ 2 & 2 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ -1 & -1 & 6 & 7 \end{bmatrix}$$

$C_2 \to C_1 - C_2$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 4 \\ 2 & 0 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ -1 & 0 & 6 & 7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_1 - R_3$, $R_4 \to R_1 + R_2$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 4 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & -5 & 11 \end{bmatrix}$$

$C_4 \to C_3 + C_4$

$$\sim \begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & -4 & -4 \\ 0 & 0 & -5 & 6 \end{bmatrix}$$

$R_3 \to -\frac{1}{4} R_3$

$$\sim \begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -5 & 6 \end{bmatrix}$$

$R_4 \to R_2 + R_4$

$$\sim \begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$$\sim \begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 5 & -1 \end{bmatrix}$$

$R_4 \to R_3 + R_4$

$$\sim \begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

$C_4 \to C_3 - C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

$R_4 \to R_4 - 6R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to 2C_2 - C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \leftrightarrow C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to \frac{1}{5} C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus, the matrix A is equivalent to the matrix $\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$

Hence, the rank is 3.

14/9/19 Practical

Q. Find the inverse of the matrix by using E-transformation.

Solⁿ:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

Solⁿ: we write as

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - R_1$, $R_2 \to R_2 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

$R_2 \to -\frac{1}{4} R_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3/4 & -1/4 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 \cancel{+} - 2R_2$, $R_3 \to R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ 3/4 & -1/4 & 0 \\ -1/4 & -1/4 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1/4 & 3/4 & -1 \\ 3/4 & -1/4 & 0 \\ -1/4 & -1/4 & 1 \end{bmatrix} A$$