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Linear Equation

Homogeneous linear equation :

Suppose
$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots a_{nm}x_n = 0 \end{array}\right\} \text{—} (1)$$

is a system of m homogeneous equation in n unknowns $x_1, x_2, \cdots x_n$.

Let
$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m\times n}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n\times 1}$$

$$O = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{n\times 1}$$

Then the system of equ^n (1) can be written in the form $AX = O$. Then the matrix A is called coefficient matrix of the system of equ^n (1).

Theorem: The no. of linearly indep^n sol^n of m homogeneous linear equ^n's in n variables, $AX = 0$, is $(n-r)$ where r is the rank of the matrix A.

Proof: Let,

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m\times n}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n\times 1}$$

Since the rank of the coefficient matrix A is ~~ree~~ r. therefore it has r linearly indep^n columns. Without loss of generality we can suppose that the 1st r columns from the left of the matrix A are linearly indep^n because it amounts only ~~two~~ ~~to~~ remaining the components of X.

Therefore the matrix A can be written as

$$A = \left[C_1 \quad C_2 \quad \cdots \quad C_n\right]_{1\times n}$$

where $c_1, c_2, \cdots c_n$ are the column vector of the matrix A.

The equ^n $AX=0$ can be now written as the vector equ^n

$$\begin{bmatrix} C_1 & C_2 & \cdots & C_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \\ x_{r+1} \\ x_n \end{bmatrix} = 0$$

$$\Rightarrow x_1 C_1 + x_2 C_2 + \cdots + x_r C_r + x_{r+1} C_{r+1} + \cdots + x_n C_n = 0 \quad \text{--- (1)}$$

Since, each of the vectors $C_{r+1}, C_{r+2}, \cdots C_n$ is a linear combination of vectors $C_1, C_2, \cdots C_r$ therefore we have a rel^n of the plane

$$\left.\begin{aligned} C_{r+1} &= P_{11} C_1 + P_{12} C_2 + \cdots + P_{1r} C_r \\ C_{r+2} &= P_{21} C_1 + P_{22} C_2 + \cdots + P_{2r} C_r \\ &\vdots \\ C_n &= P_{k1} C_1 + P_{k2} C_2 + \cdots + P_{kr} C_r \end{aligned}\right\} \text{--- (2)}$$

where $k = n-r$

Then the rel^n (2) can be written in the form

$$\left.\begin{aligned} &P_{11} C_1 + P_{12} C_2 + \cdots + P_{1r} C_r - 1 \cdot C_{r+1} + 0 \cdot C_{r+2} + \cdots + 0 C_n = 0 \\ &P_{21} \cdot C_1 + P_{22} C_2 + \cdots + P_{2n} C_r + 0 \cdot C_{r+1} - 1 \cdot C_{r+2} + \cdots + 0 C_n = 0 \\ &\vdots \\ &P_{k1} C_1 + P_{k2} C_2 + \cdots + P_{kr} C_r + 0 C_{r+1} + 0 \cdot C_{r+2} + \cdots - 1 C_n = 0 \end{aligned}\right\} \text{--- (3)}$$

Comparing (1) & (3)

$$X_1 = \begin{pmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{1r} \\ -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad X_2 = \begin{pmatrix} p_{21} \\ p_{22} \\ \vdots \\ p_{2r} \\ -1 \\ \vdots \\ 0 \end{pmatrix} \cdots, \quad X_{n-r} = \begin{pmatrix} p_{k1} \\ p_{k2} \\ \vdots \\ p_{kr} \\ 0 \\ 0 \\ \vdots \\ -1 \end{pmatrix}$$

are $(n-r)$ sol^{n}s of the equ^n $AX=0$

The vectors $X_1, X_2, \cdots X_{n-r}$ form a linearly indep^n set. For if we have a rel^n of the type $l_1 X_1 + l_2 X_2 + \cdots + l_{n-r} X_{n-r} = 0$, — (4) then comparing the $(r+1)^{th}$, $(r+2)^{th}$ --- n^{th} components on both sides of equ^n (4) we get

$-l_1 = 0, \; -l_2 = 0, \; \cdots\cdots -l_{n-r} = 0$

i.e, the vectors $X_1, X_2, \cdots X_{n-r}$ are linearly indep^n. ~~It co~~

It can be easily seen that every sol^n of the equ^n $A.X = 0$ is sum suitable linear combination of these $(n-r)$ sol^{n}s.

Suppose the vector X with components $x_1, x_2, \cdots x_n$ is any sol^n of the equ^n $AX=0$. Then the vector

$$X + x_{r+1} X_1 + x_{r+2} X_2 + \cdots + x_n X_{n-r} \quad \text{—— (5)}$$

which being a linear combination of sol^n is also a sol^n. It is quite obvious that the last $(n-r)$ components of vector ⑤ are equal to zero.
Let $z_1, z_2, \cdots z_r$ be the 1st r components of the vector ⑤ then the vectors whose components are $(z_1, z_2, \cdots z_r, 0, 0, \cdots 0)$ is a sol^n of the equ^n $AX = 0$.

Therefore, from equ^n ① we have

$$z_1 C_1 + z_2 C_2 + \cdots + z_r C_r = 0$$

But the vectors $C_1, C_2, \cdots C_r$ are linearly indep^n. therefore we have $z_1 = 0, z_2 = 0, \cdots z_r = 0$

Hence, the vector ⑤ is a zero vector.

$$\therefore X = -x_{r+1} X_1 - x_{r+2} X_2 - \cdots - x_n X_n$$

Thus, every sol^n X is a linear combination of the $(n-r)$ linearly indep^n sol^{n}s $X_1, X_2, \cdots X_{n-r}$

$\therefore$ The set of sol^{n}s $\{X_1, X_2, \cdots X_{n-r}\}$ forms a basis of the vector space of all the sol^{n}s of the system of equ^n $AX = 0$

proved//

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Some important conclusion about the nature of the solution of the equation $AX = 0$

Suppose we have ~~an eq~~ m equ^n in n unknowns then the co-efficient matrix A will be of the type $m \times n$. Let, r be the rank of the matrix A obviously r can not be greater than n. Therefore we have either $r = n$, $r < n$.

Case I :

If $r = n$, the equ^n $AX = 0$ will have no linear indep^n sol^n. In this case the zeros sol^n will be only ~~case~~

Case II :

If $r < n$, ~~will~~ we shall have $(n - r)$ linearly indep^n sol^n. Thus, in this case the equ^n $AX = 0$ will have infinite no. sol^n.

Case III :

Suppose $m < n$ i.e, the no. of sol^{n}s is less than the no. of unknowns since $r \leq m$, therefore r is definatly less than n. Hence in this case the given system of equ^n must posses a non zero sol^n. The no. of sol^n of the equ^n $AX = 0$ will be infinite

Def^n : Define fundamental system of equ^n of $AX = 0$.

Ans -> A set of linearly indep^n sol^n $X_1 X_2 \cdots X_k$ of the system of homogeneous equ^n $AX=0$ is called the fundamental system of eq^n $AX=0$, if every sol^n of $AX=0$ can be written as a linear combination of the vectors in the form $X = C_1 X_1 + C_2 X_2 + \cdots + C_n X_n$

where, $C_1, C_2 \cdots C_n$ are suitable unknown

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Working group for finding the solution of the equation $AX = 0$:

Reduce the coefficient matrix A to echelon form by applying elementary row transformation only. This echelon form will help us to know the rank of the matrix A. Suppose the matrix A is of the type $m \times n$ and its rank comes out to be r. If $r < m$, then in the process of reducing the matrix A to echelon form, $(m-r)$ equations will be eliminated. The given system of m equations will thus be ~~of~~ replaced by an equivalent system of r equations. Solving these r equations (by Cramer's rule or otherwise), we can express the values of some r unknowns in terms of the remaining $n-r$ unknowns. Thus $n-r$ unknowns can be given any arbitrarily chosen values.

If $r=n$, the zero solution (trivial solution) will be the only solution. If $r<n$, there will be an infinity of solutions.

Q Solve completely the system of equ^n.

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0$$

Sol^n: The given system of equ^n is equivalent to single matrix equ^n.

$$AX = 0$$

$$\begin{pmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$\begin{pmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$R_3 \to R_3 - 2R_2$
$-14 - (-14)$

~~1 3 -~~

$R_3 \to R_3 - 2R_2$

$$\begin{pmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

~~The coefficient matrix~~

The rank of the coefficient matrix A is 2

The given system of equ^n has $(3-2) = 1$ linearly indep^n sol^n.

$x + 3y - 2z = 0$ —— ①

$-7y + 8z = 0$ —— ②

$\Rightarrow y = \frac{+8z}{+7} = \frac{8z}{7}$

① $\Rightarrow x + 3\left(\frac{8z}{7}\right) - 2z = 0$

$\Rightarrow x + \frac{24z}{7} - 2z = 0$

$\Rightarrow x + \frac{24z - 14z}{7} = 0$

$\Rightarrow x + \frac{10z}{7} = 0$

$\Rightarrow x = -\frac{10z}{7}$

Hence, $x = \frac{-10z}{7}$, $y = \frac{8z}{7}$, $z = c$

constitute the general sol^n of the given system.

$\therefore x = \frac{-10c}{7}$, $y = \frac{8c}{7}$, $z = c$.

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Q Solve completly the system of equ^n

$x + y + z = 0$

$2x - y - 3z = 0$

$3x - 5y + 4z = 0$

$x + 17y + 4z = 0$

P. T. O.

Solⁿ: The given system of equⁿ is equivalent to single matrix equⁿ

$$AX = 0$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & -8 & 1 \\ 0 & 16 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$R_4 \to R_4 + 2R_3$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & -8 & 1 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$R_3 \to 3R_3$, $R_4 \to 3R_4$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & -24 & 3 \\ 0 & 48 & 9 \end{pmatrix}$$

$R_3 \to R_3 + 8R_2$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & 0 & 43 \\ 0 & 48 & 9 \end{pmatrix}$$

$R_3 \to R_3 + 16R_2$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & 0 & 43 \\ 0 & 0 & -71 \end{pmatrix}$$

$R_4 \to R_4 + \frac{71}{43} R_3$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & 0 & 43 \\ 0 & 0 & 0 \end{pmatrix}$$

~~The above coefficient matrix A.~~

The rank of the coefficient matrix A is 3 & the no. of unknowns is also 3. ~~B~~ Therefore rank A is equal to the no. of unknowns.

∴ The given system of equ^n posseses zero sol^n

Hence, zero sol^n i.e, $x=0$, $y=0$, $z=0$ is the only sol^n of the given system of equ^n.

Practical

Q Find all the sol^n of the following system of equ^n

$$3x + 4y - z - 6W = 0$$
$$2x + 3y + 2z + 3W = 0$$
$$2x + y - 14z - 9W = 0$$
$$x + 3y + 13z + 3W = 0.$$

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P.T.O.

Solⁿ: The given system of equⁿ is equivalent to single matrix equⁿ

$$AX = 0$$

$$\begin{bmatrix} 3 & 4 & -1 & -6 \\ 2 & 3 & 2 & -3 \\ 2 & 1 & -14 & -9 \\ 1 & 3 & 13 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 3 & 13 & 3 \\ 2 & 3 & 2 & -3 \\ 2 & 1 & -14 & -9 \\ 2 & 4 & -1 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - 3R_1$

$$\begin{bmatrix} 1 & 3 & 13 & 3 \\ 0 & -3 & -24 & -9 \\ 0 & -5 & -40 & -15 \\ 0 & -5 & -40 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 3 & 13 & 3 \\ 0 & -3 & -24 & -9 \\ 0 & -5 & -40 & -15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to -\frac{1}{3}R_2$, $R_3 \to -\frac{1}{5}R_3$

$$\begin{bmatrix} 1 & 3 & 13 & 3 \\ 0 & 1 & 8 & 3 \\ 0 & 1 & 8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 3 & 13 & 3 \\ 0 & 1 & 8 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The rank of A is obviously 2 which is less than the no. of unknowns 4. Therefore the given system of equ^{n}s possesses 4-2 i.e, 2 linearly indep^n sol^{n}s.
The given system of equ^{n}s is equivalent to the equ^n

$$\begin{bmatrix} 1 & 3 & 13 & 3 \\ 0 & 1 & 8 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus the given system of four equ^{n}s is equivalent to the system of two equ^{n}s, i.e,

$$\left.\begin{aligned} x + 3y + 13z + 3w &= 0 \\ y + 8z + 3w &= 0 \end{aligned}\right\}$$

From these equ^{n}s we get

$y = -8z - 3w$, $x = -3(-8z - 3w) - 13z - 3w$

i.e, $y = -8z - 3w$, $x = 11z + 6w$.

Hence, $x = 11C_1 + 16C_2$, $y = -8C_1 - 3C_2$, $z = C_1$, $w = C_2$ constitute the general sol^n of the given system of equ^{n}s, where C_1 & C_2 are arbitary no.s.
Since we can give any ~~arbitary~~ arbitrary values to C_1 & C_2 therefore the given system of equ^n has an

infinite no. of sol^{n}s.

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System of linear non-homogeneous equations:

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Theorem: The system of equ^{n}s $AX=B$ is consistent i.e, possesses a sol^n, iff the coefficient matrix A and the augemented matrix $[AB]$ are of the same rank.

Proof: Let, $C_1, C_2, \cdots, C_n$ denote the column vectors of the matrix A. The equation $AX=B$ is then equivalent to

$$[C_1\, C_2 \cdots C_n]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = B$$

$$\text{i.e, } x_1C_1 + x_2C_2 + \cdots + x_nC_n = B \quad \text{—— (1)}$$

Let now r be the rank of the matrix A. The matrix A has then r linearly indep^n columns &, without loss of generality, we can suppose that the 1st r columns $C_1, C_2, \cdots, C_r$ form a linearly indep^n set so that each of the remaining $n-r$ columns is a linear combination of these r columns.

<u>The condition is necessary</u>: If the given system of equ^{n}s is consistent, there must exist n scalars (numbers) $k_1, k_2, ---, k_n$ such that

$$k_1C_1 + k_2C_2 + \cdots + k_nC_n = B \quad \text{——} \quad (2)$$

Since each of the $n-r$ columns $C_{r+1}, C_{r+2}, ---, C_n$ is a linear combination of the 1st r columns $C_1, C_2, --C_r$ it is obvious from (2) that B is also a linear combination of $C_1, C_2, ---C_r$. Thus the max^m no. of linearly indep^n columns of matrix $[AB]$ is also r. Therefore, the matrix $[AB]$ is also of rank r. Hence the matrices A & $[AB]$ are of the same rank.

<u>The condition is sufficient</u>: Now suppose that the matrices A and $[AB]$ are of the same rank r. The max^m no. of linearly indep^n columns of the matrix $[AB]$ is then r. But the 1st r columns $C_1, C_2, --- C_r$ of the matrix $[AB]$ already form a linearly indep^n set. Therefore the column B should be expressed as a linear combination of the columns $C_1, C_2, --- C_r$.

Thus, $\exists$ r scalars $k_1, k_2, --- k_r$ such that

$$k_1C_1 + k_2C_2 + --- k_rC_r = B \quad \text{——} \quad (3)$$

Now, (3) may be written as

$$k_1C_1 + k_2C_2 + \cdots + k_rC_r + 0.C_{r+1} + 0.C_{r+2} + \cdots + 0.C_n = B \quad \text{——} \quad (4)$$

Comparing (1) & (4), we see that

$$x_1 = k_1, x_2 = k_2, ---, x_r = k_r, x_{r+1} = 0, x_{r+2} = 0, ---, x_n = 0$$

constitute a sol^n of the equ $AX = B$.

Therefore the given system of equ^{n}s is consistent.

Theorem: If A be a n-rowed non-singular matrix, X be a $n \times 1$ matrix, B be an $n \times 1$ matrix, the system of equ^{n}s $AX = B$ has a unique solution.

Proof: If A be a n-rowed non-singular matrix, the rank of the matrices A & $[AB]$ are both n.

Therefore, the system of equ^n $AX = B$ is consistent i.e, possesses a unique sol^n.

Premultiplying both sides by $AX = B$ by A^{-1}, we have

$$A^{-1}(AX) = A^{-1}B$$

$$\Rightarrow IX = A^{-1}B$$

$$\Rightarrow X = A^{-1}B$$

is a sol^n of the equ^n $AX = B$.

To show that the sol^n is unique, let us suppose that X_1 & X_2 be two sol^{n}s of $AX = B$.

Then, $AX_1 = B$, $AX_2 = B$

$$\Rightarrow AX_1 = AX_2$$

$$\Rightarrow X_1 = X_2$$

Hence, the sol^n is unique.

Working rule for finding the solution of the equation ~~A~~ $AX = B$:

Suppose the coefficent matrix A is of the type $m \times n$ i.e, we have m equation in n unknowns. The augmented matrix $[AB]$ should be reduced it to a echelon form by applying only E-rowed transform on it. This echelon form will anable us to know the ranks of the augmented matrix $[AB]$ & the coefficient of the matrix A. then, the following different caseses arrise.

Case I : ~~rank~~ $\rho(A) < \rho[AB]$

In this case, the equ^n $AX = B$ are inconsistant i.e, they have no sol^n.

Case II : $\text{Rank}(A) = \text{Rank}[AB] = r$ (say)

In this case, the equ^n $AX = B$ are consistant i.e, they posses a sol^n.

If $r < m$ then ~~a~~ in the process of reducing the matrix $[AB]$ to echelon form, $(m-r)$ equ^n will be elemenated. The given system of m equ^{n}s will be replaced by an ~~by~~ equivalent system of r equ^n. From these r equ^{n}s we shall be able to expressed the values of some r unknowns in terms of remaning $(n-r)$ unknowns.

① If $r=n$ then $n-r=0$, so that no variable is to be assined arbitary values & therefore in these case there will be a unique sol^n.

② If $r<n$ then $(n-r)$ variable can be assined arbitarey values, in these case there will be an infinite no. of sol^n.

③ If $m<n$ then $r \leq m \leq n$. Thus in these case, $n-r<0$. Therefore when the no. of equ^n is less than the no. of unknowns the equ^n will have an infinite no. of sol^n, provided they are consistant.

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Q. Show that the equ^{n}s

$$x+y+z+3=0$$

$$3x+y-2z+2=0$$

$$2x+4y+7z-7=0$$

are not consistant.

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Sol^n: The given equ^n can be written as

$$x + y + z = -3$$
$$3x + y - 2z = -2$$
$$2x + 4y + 7z = 7$$

The given system of equ^n is equivalent to singal matrix equ^n

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}$$

$\therefore$ The augmanted matrix

$$[AB] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & -3 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 7 & 7 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 2 & 5 & 13 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 0 & 0 & 20 \end{array}\right]$$

Above is the echelon form of the matrix [AB].

Therefore we have rank $[AB] = 3$ ~~ab~~

Also, by the same E-row transformation

we get

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank $A = 2$

Since, rank $A \neq$ rank $[A B]$ therefore the given system of equ^{n}s are ~~..~~ not consistant i.e, they have no sol^n.

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