*Dynamic Stability of Market price :

In a partial equilibrium model of market, price and quantity are shown to be in equilibrium. But the equilibrium price changes over time (t) in proportion to change in excess demand and therefore we may not have a stable price in the short-run. However, the market price will be stable in the long-run depending on certain conditions. The solution of differential equation enables us to obtain the conditions for dynamic stability of equilibrium. Let us consider a single commodity market model,

$$Q_d = a - bP \quad (a, b > 0) \qquad (1)$$

$$Q_s = -c + dP \quad (c, d > 0) \qquad (2)$$

$$Q_s = Q_d \qquad (3)$$

The equilibrium price is given by

$$\bar{P} = \frac{a+c}{b+d}$$ [∵ Substituting the values of equation (1) and (2) into (3)]

If the initial price $P(0)$ is exactly equal to the equim price, the market price is stable and there is no need of any dynamic analysis of prices.

But if the initial price is different from the equi^m price, i.e., if $P(0) \neq \bar{P}$ then market will be unstable as there will be divergence between demand and supply at a given price. The price will be stabilised through a process of adjustment over time, during that adjustment process, Q_d and Q_s will also change over time as they are functions of price (P).

So, the question is whether market price (P_t) tends to converge to equi^m price, $\bar{P}$, when $t \rightarrow \infty$?

To test the convergence of P_t to $\bar{P}$, we are to find out the time path of P_t. For that, we must know the pattern of price change.

We know that change in price is governed by relative strength of demand and supply. So we can consider the change in price over time is directly proportional to excess demand $(Q_d - Q_s)$ such that,

$$\frac{dP}{dt} = \alpha (Q_d - Q_s) \quad (\alpha > 0) \qquad (4)$$

where α represents the adjustment co-efficient which remains constant over time.

Now substituting the values of Q_d and Q_s into equation (4),

$$\frac{dP}{dt} = \alpha [a - bP + c - dP]$$

$$= \alpha (a + c) - \alpha (b + d) P$$

or, $$\frac{dP}{dt} + \alpha (b + d) P = \alpha (a + c) \qquad (5)$$

equation (5) represents a first order differential equation with constant co-efficient and a constant term. To get the time ~~time~~ path P_t, we are to find out the solution of the differential equation.

Following the general solution of differential equation $\left(\frac{dy}{dx} + ay = b\right)$, as

$$y(x) = \left[y(0) - \frac{b}{a}\right] e^{-ax} + \frac{b}{a}$$

we obtain the solution of (5) as

$$P_t = \left[P_0 - \frac{\alpha(a+c)}{\alpha(b+d)}\right] e^{-\alpha(b+d)P} + \frac{\alpha(a+c)}{\alpha(b+d)}$$

or, $$P_t = \left[P_0 - \bar{P}\right] e^{-\beta t} + \bar{P} \qquad (6)$$

where, $\bar{P} = \frac{a+c}{b+d}$ and $\beta = \alpha(b+d) > 0$

Since α, b and d are positive

From the time path of P_t in equation (6), it appears that regarding whether the initial price $P(0)$ is greater than equim price $\bar{P}$ or less than $\bar{P}$, P_t will tend to $\bar{P}$ when $t \to \infty$. This is because of the fact that $\beta > 0$ and so $e^{-\beta t} = \frac{1}{e^{\beta t}}$ will tend to zero as $t \to \infty$. Thus, finally the time path of P_t will converge to the level of $\bar{P}$ and the equim is said to be dynamically stable.

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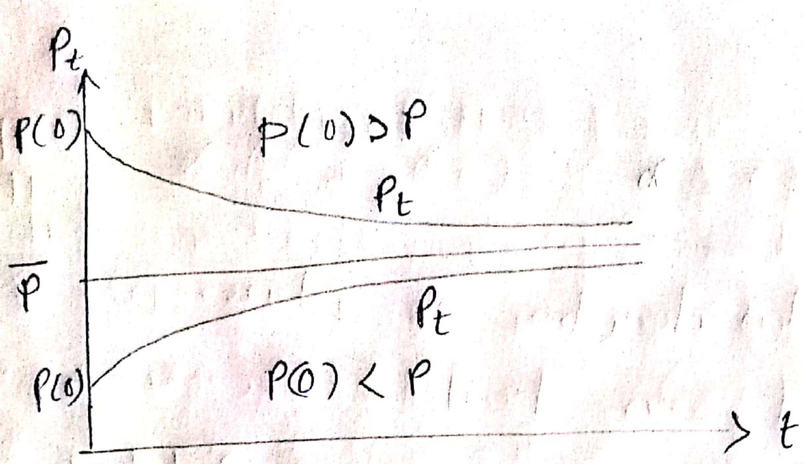

As shown in the above figure, when the initial price is greater than the equilibrium price $P(0) > \overline{P}$, the current price P_t approaches $\overline{P}$ when $t \to \infty$. Similarly, when the initial price is less than the equilibrium price, $P(0) < \overline{P}$, even then the current price P_t will approach $\overline{P}$ as $t \to \infty$. Finally, if $P(0) = \overline{P}$ then $P_t = \overline{P}$.