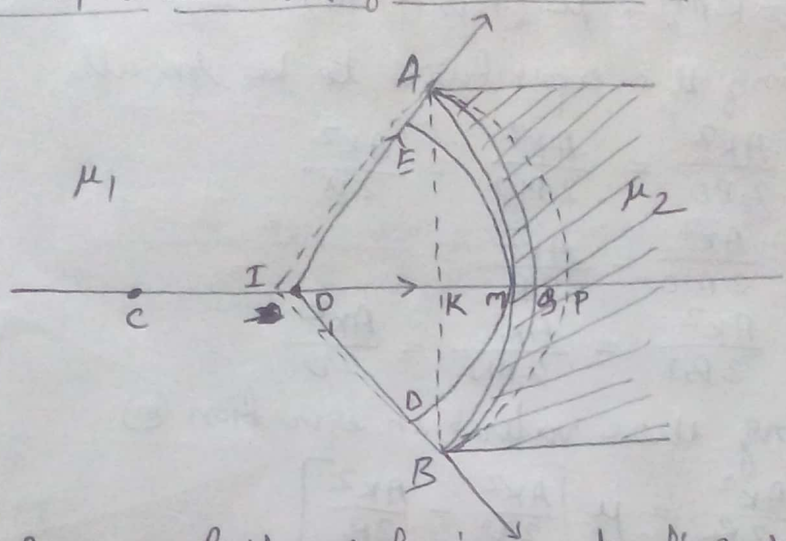
Huygen's Wave theory

Refraction of a spherical wavefront at a spherical surface:



Let AMB represent the spherical refracting surface separating the media of refractive indices μ_1 and μ_2. C is the centre of ~~the~~ curveture of the refracting surface and R is the radius of curveture. EMD is the incident spherical wavefront. In the absence of the surface AMB the wave must have advanced a distance $MP = EA = DB = v_1 t$ in time 't', where v_1 is the velocity of light in the first medium. By the time the disturbance at the points E and D reaches the points A and B, the secondary waves from M must have travelled a distance $MQ = v_2 t$ in the same time, where v_2 is the velocity of light in the second medium. Therefore, AQB forms the refracted spherical wavefront whose centre of curveture is I. Here $MO = u$ the object distance and $MI = v$, the image distance, ~~AQB is~~

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AKB is perpendicular to the axis

$$MP = v_1 t \; ; \; MB = v_2 t$$

$$\frac{MP}{MB} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1} = \mu \quad \text{——— (i)}$$

where μ is the refractive index of the second medium with respect to the first medium

$$MP = \mu MB \quad \text{——— (ii)}$$

Thus, the effect of refraction is to reduce the curvature of the refracted wavefront when $\mu_2 > \mu_1$ and to increase the curvature when $\mu_2 < \mu_1$.

$$MP = KP - KM$$

$$MB = KB - KM$$

substituting these values in eq^n (ii)

$$(KP - KM) = \mu (KB - KM) \quad \text{——— (iii)}$$

considering the aperture to be small.

$$KP = \frac{AK^2}{2PO} = \frac{AK^2}{2MO} = \frac{AK^2}{2u}$$

$$KM = \frac{AK^2}{2MC} = \frac{AK^2}{2R}$$

$$KB = \frac{AK^2}{2BI} = \frac{AK^2}{2MI} = \frac{AK^2}{2v}$$

substituting these values in equation (iii)

$$\frac{AK^2}{2u} - \frac{AK^2}{2R} = \mu \left[\frac{AK^2}{2v} - \frac{AK^2}{2R} \right]$$

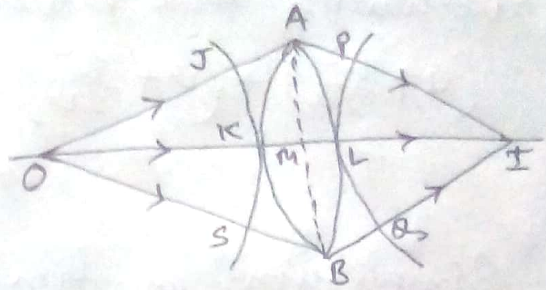
$$\frac{1}{u} - \frac{1}{R} = \frac{\mu}{v} - \frac{\mu}{R}$$

$$\frac{\mu - 1}{R} = \frac{\mu}{v} - \frac{1}{u}$$

Here u, v and R are all $-ve$

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R} \quad \text{——— (iv)}$$

Refraction through a convex lens:



AB is the aperture of the lens. Let μ be the refractive index of the material of the lens. 'O' is an object point on the lens axis and I is the image. Let JKS be the incident spherical wavefront. By the time the disturbance at the points J and S reaches the points P and Q, the secondary waves from the point K must have travelled a distance KL through the medium of the lens. Therefore, PLQ forms the refracted spherical wavefront whose centre of curvature is I. Hence I is the image of 'O'.

Also, the optical path

$$OA + AI = OK + \mu KL + LI$$

$$= (OM - KM) + \mu(KM + ML) + (MI - ML) \quad \text{—— (i)}$$

AMB is perpendicular to the axis of the lens

Here $AM = h$; $MO = u$, $MI = v$ and R_1 and R_2 are the radii of curvature of the first and second surface AKB and ALB of the lens.

In the ΔOAM

$$OA^2 = OM^2 + AM^2 = OM^2\left[1 + \frac{AM^2}{OM^2}\right]$$

$$OA = OM\left[1 + \frac{AM^2}{OM^2}\right]^{1/2} = u\left[1 + \frac{h^2}{u^2}\right]^{1/2} = u + \frac{h^2}{2u} \text{ (approx)} \quad \text{— (ii)}$$

Similarly from ΔAMI

$$AI^2 = MI^2 + AM^2 = MI^2\left[1 + \frac{AM^2}{MI^2}\right]$$

$$AI = MI\left[1 + \frac{AM^2}{MI^2}\right]^{1/2} = v\left[1 + \frac{h^2}{v^2}\right]^{1/2} = v + \frac{h^2}{2v} \text{ (approx)} \quad \text{— (iii)}$$

For the spherical surface AKB

$$KM = \frac{h^2}{2R_1} \quad \text{—— (iv)}$$

and for the spherical surface ALB

$$LM = \frac{h^2}{2R_2} \quad \text{—— (v)}$$

$$\therefore KM + LM = KL = \frac{h^2}{2}\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \quad \text{—— (vi)}$$

substituting these values in equation(i)

$$\left(u+\frac{h^2}{2u}\right)+\left(v+\frac{h^2}{2v}\right)=\left\{\left(\mu-\frac{h^2}{2R_1}\right)+\mu\left(\frac{h^2}{2R_1}+\frac{h^2}{2R_2}\right)+\left(v-\frac{h^2}{2R_2}\right)\right\} \quad \text{(vii)}$$

Simplifying equation (vii)

$$\frac{h^2}{2v}+\frac{h^2}{2u}=\mu\left[\frac{h^2}{2R_1}+\frac{h^2}{2R_2}\right]-\left[\frac{h^2}{2R_1}+\frac{h^2}{2R_2}\right]$$

$$\frac{1}{v}+\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

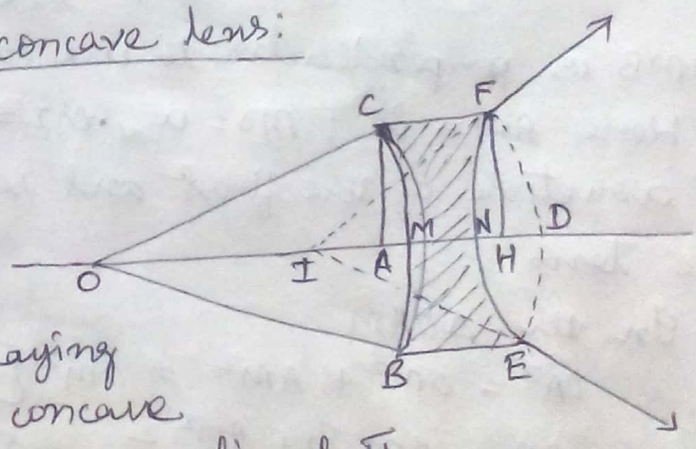
According to the sign convention u is -ve, v is +ve, R_1 is +ve and R_2 is -ve.

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{(viii)}$$

If the object is at infinity $u=\infty$ and $v=f$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{(ix)}$$

Refraction through a concave lens:



consider a point object 'O' laying on the principal axis of concave lens and suppose CAB is the portion of the wavefront which touches the lens surface at C and B. During the time the disturbance reaches from C to F in the medium, the disturbance starting from A will reach D partly traveling through air and partly through the medium. As the thickness of the lens in the middle is small and velocity of the wave c_1 in air is greater than the velocity c_2 in the medium, the refracted wavefront will take the form EDF and will appear to originate from I. Hence I will be the virtual image of the object 'O'.

Draw CG and FH perpendiculars on the axis and let the length of each perpendicular be denoted by y.

Now the time taken by the disturbance to travel in the medium from C to F is given by $\frac{CF}{c_2}$. The time taken by disturbance to travel from A to D partly in air and partly in the medium

$$= \frac{AM}{c_1} + \frac{MN}{c_2} + \frac{ND}{c_1}$$

$$\frac{CF}{c_2} = \frac{AM}{c_1} + \frac{MN}{c_2} + \frac{NH}{c_1}$$

Now, $CF = GH = GM + MN + NH$

Hence $\frac{GM}{c_2} + \frac{MN}{c_2} + \frac{NH}{c_2} = \frac{AM}{c_1} + \frac{MN}{c_2} + \frac{ND}{c_1}$

Multiplying both sides by c_1, we get

$$\frac{c_1}{c_2} GM + \frac{c_1}{c_2} NH = AM + ND$$

$$\mu GM + \mu NH = AM + ND$$

$$\mu GM + \mu NH = GM - AG + NH + HD$$

$$GM(\mu - 1) + NH(\mu - 1) = -AG + HD$$

$$HD - AG = (\mu - 1)(GM + NH) \quad \text{———} \quad (iii)$$

If R_1 and R_2 are the radii of curvatures of the surfaces CMB and FNE respectively and $OA = u$ and $IA = v$, then

$$GM = \frac{y^2}{2R_1};\ NH = \frac{y^2}{2R_2};\ AG = \frac{y^2}{2u} \text{ and } HD = \frac{y^2}{2v}$$

substituting these values in (iii) we have

$$\frac{y^2}{2v} - \frac{y^2}{2u} = (\mu-1)\left(\frac{y^2}{2R_1} + \frac{y^2}{2R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = (\mu-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

According to the sign convention u, v and R_1 are negative and R_2 is positive.

$$-\frac{1}{v} + \frac{1}{u} = (\mu-1)\left(-\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the object lies at infinity, the image is formed at the focus of the lens.

$$\therefore \frac{1}{u} = \frac{1}{\infty} = 0 \quad \text{and} \quad \frac{1}{v} = \frac{1}{f}$$

Hence $\quad \frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

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