

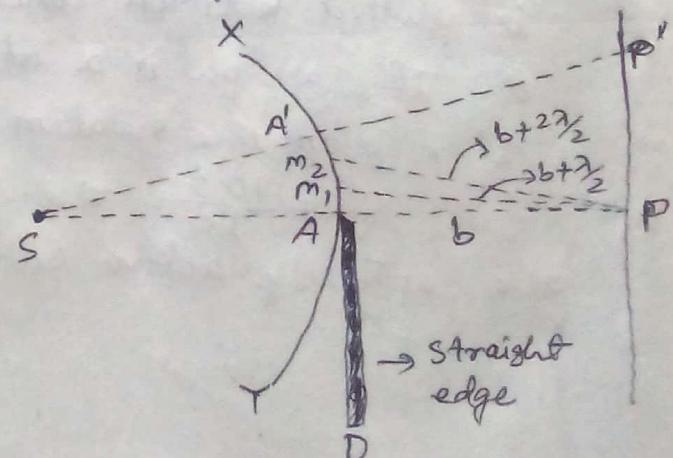
Fresnel and Fraunhofer Diffraction:

Diffraction phenomena can conveniently be divided into two groups viz. (i) Fresnel diffraction phenomena and (ii) Fraunhofer diffraction phenomena. In the Fresnel class of diffraction, the source or the screen or both are at infinite distances from the aperture or obstacle causing diffraction. In this case, the effect at a specific point on the screen due to the exposed incident wavefront is considered and no modification is made by lenses and mirrors. In such a case, the phenomenon observed on the screen is called Fresnel diffraction pattern. In the Fraunhofer class of diffraction phenomena, the source and the screen on which the pattern is observed are at infinite distances from the aperture or the obstacle causing diffraction. Fraunhofer diffraction pattern can be easily observed in practice. The incoming light is rendered ~~per~~ parallel with a lens and the diffracted beam is focused on the screen with another lens. Observation of Fresnel diffraction phenomena don't require any lenses. Theoretical treatment of Fraunhofer diffraction phenomena is simpler. Fresnel class of diffraction phenomena are

Fresnel diffraction due to a straight edge

Let S be narrow slit illuminated by a source of monochromatic light of wavelength λ . The length of the slit is perpendicular to the plane of the paper. AD is the straight edge and the length of the edge is parallel to the length of the slit.

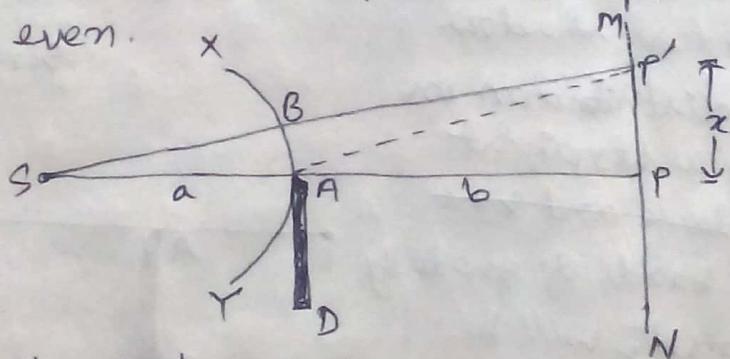
XY is the incident cylindrical wavefront.



P is a point on the screen and SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. Below the point 'P' is the shadow geometrical shadow and above P is the illuminated portion. XY is the wavefront, A is the pole of the wavefront and AM₁, M₁M₂ etc. measure the thickness of the 1st, 2nd, 3rd etc. half period strips. With reference in the order of the strip, the area of the strip decreases.

$$AP = b \quad ; \quad PM_1 = b + \frac{1}{2} \quad ; \quad PM_2 = b + 2\frac{1}{2} \text{ etc.}$$

Let P' be a point on the screen in the illuminated portion. To calculate the resultant effect at P' due to the wavefront XY, join S to P'. This line meets the wavefront at B. B is the pole of the wavefront with reference to the point P' and the intensity at P' will depend mainly on the number of half period strips enclosed between the points A and B. The effect at P' due to the wavefront above B is the same at all points on the screen whereas it is different at different points due to the wavefront between B and A. The point P' will be of maximum intensity, if the number of half period strips enclosed betn B and A is odd and the intensity at P' will be minimum if the number of half period strips enclosed between B and A is even.



Let the distance between the slit and the straight edge be 'a' and the distance between the straight edge and the screen 'b'. Let PP' be 'x'

The path difference $\delta = AP' - BP'$

$$\delta = (b^2 + x^2)^{1/2} - [SP' - SB]$$

$$= (b^2 + x^2)^{1/2} - \sqrt{(a+b)^2 + x^2} - a$$

$$= b \left[1 + \frac{x^2}{2b^2} \right] - (a+b) \left[1 + \frac{x^2}{2(a+b)^2} \right] + a$$

$$\begin{aligned}
 &= b + \frac{x^2}{2b} - a - b - \frac{x^2}{2(a+b)} + a \\
 &= \frac{x^2}{2} \left(\frac{1}{b} - \frac{1}{a+b} \right) \\
 &= \frac{x^2}{2} \left(\frac{a+b-b}{a(a+b)} \right) \\
 \delta &= \frac{x^2}{2} \frac{a}{b(a+b)}
 \end{aligned}$$

The point p' will be of maximum intensity if $\delta = (2n+1)\frac{\lambda}{2}$

$$\therefore (2n+1)\frac{\lambda}{2} = \frac{ax_n^2}{2b(a+b)}$$

$$x_n^2 = \frac{(2n+1)(a+b)b\lambda}{a}$$

$$x_n = \sqrt{\frac{(2n+1)(a+b)b\lambda}{a}} \quad \text{(i)}$$

where x_n is the distance of the n^{th} bright band from P

Similarly p' will be of minimum intensity if $\delta = 2n\frac{\lambda}{2}$

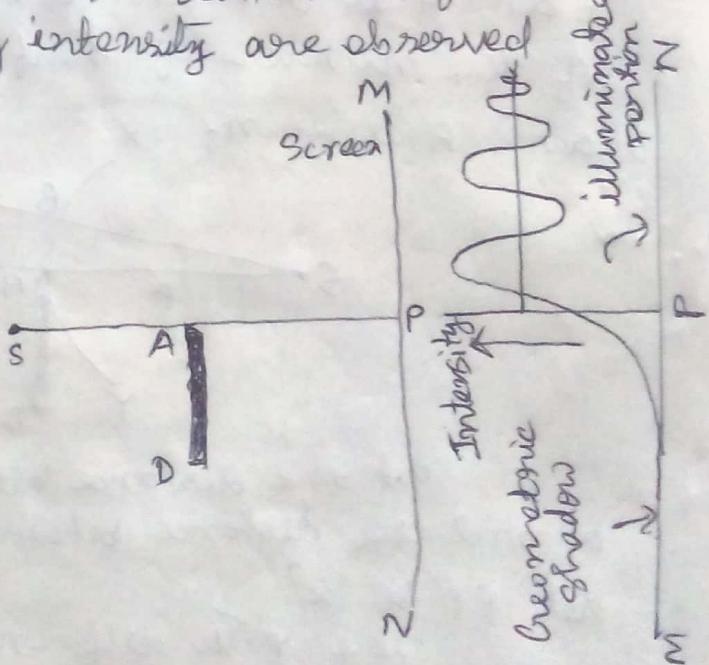
$$\therefore 2n\frac{\lambda}{2} = \frac{ax_n^2}{2b(a+b)}$$

$$x_n^2 = \frac{2n(a+b)b\lambda}{a}$$

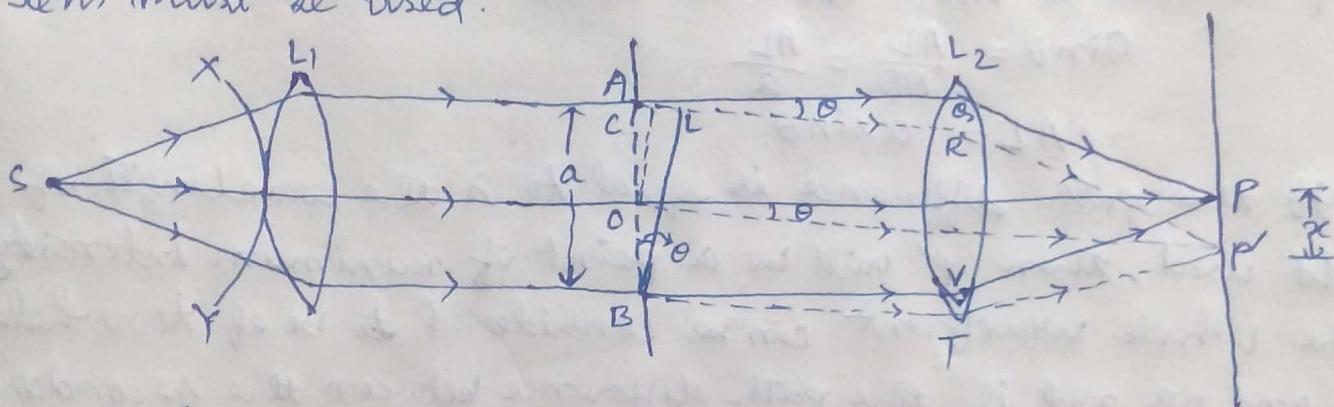
$$x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}}$$

where x_n is the distance of the n^{th} dark band from P , thus diffraction bands of varying intensity are observed above the geometrical shadow.

The intensity distribution on the screen due to a straight edge. Alternating dark and bright bands of gradually diminishing intensity will be observed and the intensity falls gradually in the region of the geometrical shadow.



Fraunhofer diffraction at a single slit: To obtain the Fraunhofer diffraction pattern, the incident wavefront must be plane and the diffracted light is collected on the screen with the help of a lens. Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.



S is a narrow slit perpendicular to the plane of paper and illuminated by monochromatic light. L_1 is the collimating lens and AB is a slit of width a . XY is the incident spherical wavefront. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN . The screen is perpendicular to the plane of the paper. The line SP is perpendicular to the screen. L_1 and L_2 are achromatic lenses.

A plane wave is incident on the slit AB and each point on this wavefront is a source of secondary disturbance.

The secondary waves traveling in the direction parallel to OP viz. AB and BV come to focus at P and a bright central image is observed. The secondary waves from points equidistant from 'O' and situated in the upper and lower halves OA and OB of the wavefront travel the same distance in reaching P and hence the path difference is zero. The secondary waves reinforce one another and P will be at a point of maximum intensity.

Now consider a secondary waves traveling in the direction AR, inclined at an angle θ to the direction OP. All the secondary wave traveling in this direction reach the point P' on the screen. The point P' will be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wavefront. Draw OC and BL perpendicular to AR. Then in $\triangle ABL$

$$\sin \theta = \frac{AL}{AB} = \frac{AL}{a}$$

$$AL = a \sin \theta$$

If the path difference is equal to λ the wavelength of light used, then P' will be a point of minimum intensity. The whole wavefront can be considered to be of two halves OA and OB and if the path difference between the secondary waves from A and B is λ , then the path difference between the secondary waves from A and O will be $\frac{\lambda}{2}$. Similarly for every point in the upper half OA, there is a corresponding point in the lower half OB, and the path difference betn the secondary waves from these points is $\frac{\lambda}{2}$. Destructive interference takes place and the point P' will be of minm intensity. In general

$$a \sin \theta_n = n\lambda$$

$$\Rightarrow \sin \theta_n = \frac{n\lambda}{a}; n=1, 2, 3, \dots \text{etc.}$$

θ_n gives the direction of the n th minimum.

$$a \sin \theta_n = (2n+1)\frac{\pi}{2}$$

$$\sin \theta_n = \frac{(2n+1)\lambda}{2a} ; n = 1, 2, 3, \dots \text{etc}$$

Thus, diffraction pattern due to a single slit consists of a central bright maximum at P followed by secondary maxima and minima on both the sides. The intensity distribution on the screen, P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points A and B is $\lambda, 2\lambda$, etc. correspond to the position of secondary minima.

